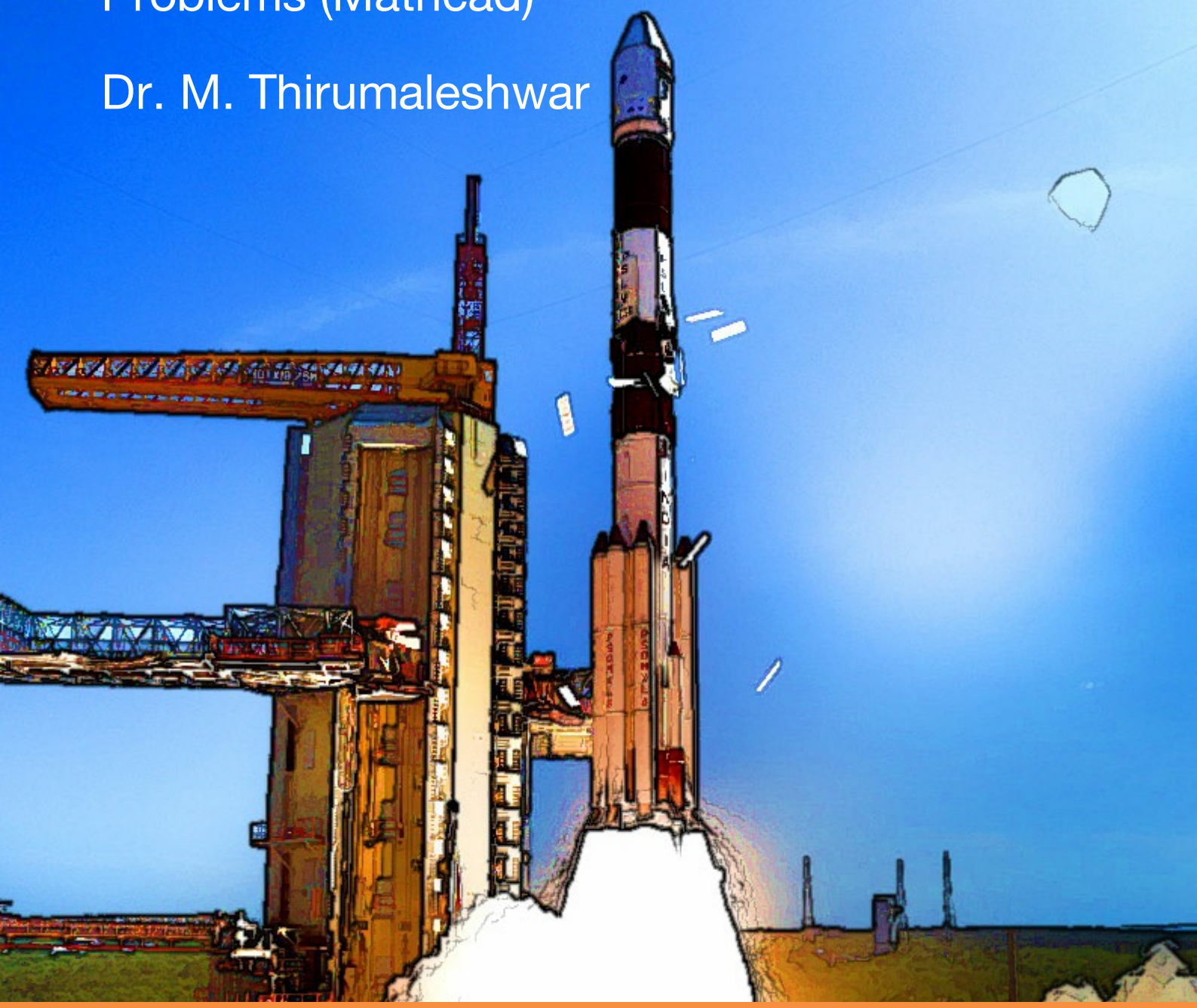
Cryog. Engineering: Software Solutions Part-III-A

Cryogenic Refrigeration systems: Theory + Problems (Mathcad)

Dr. M. Thirumaleshwar



bookboon.com
The eBook company

DR. M. THIRUMALESHWAR

CRYOGENIC ENGINEERING: SOFTWARE SOLUTIONS PART-III-A

CRYOGENIC REFRIGERATION SYSTEMS – THEORY + PROBLEMS (MATHCAD)

Download free eBooks at bookboon.com

Cryogenic Engineering: Software Solutions Part-III-A: Cryogenic Refrigeration systems – Theory + Problems (Mathcad)
1st edition
© 2017 Dr. M. Thirumaleshwar & bookboon.com
ISBN 978-87-403-1752-7
Peer review by Dr. Thirumaleshwara Bhat, Principal, SMVITM

Download free eBooks at bookboon.com

CONTENTS

	Dedication	**6**
	Preface	**7**
	About the Author	**10**
	About the Software used	**12**
4	**Cryogenic Refrigeration systems**	**13**
4.1	Definitions, Statements and Formulas used[1-9]:	14
4.2	Problems solved with Mathcad:	88
4.3	References	233

Deloitte.

Discover the truth at www.deloitte.ca/careers

© Deloitte & Touche LLP and affiliated entities.

Download free eBooks at bookboon.com

Click on the ad to read more

Cryogenic Engineering: Software Solutions Part-III-B:
Cryogenic Refrigeration systems – Problems (EES)

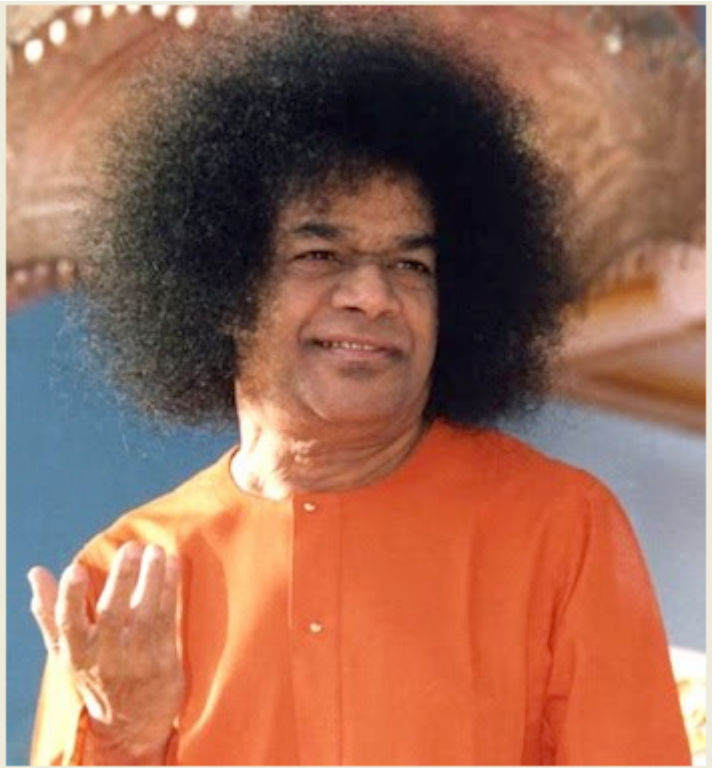
4	**Cryogenic Refrigeration systems: Problems (EES)**	**Part-III-B**
4.1	Definitions, Statements and Formulas used[1-9]: See Part-III A	Part-III-B
4.2	Problems solved with Mathcad … See Part-III A	Part-III-B
4.3	Problems solved with EES	Part-III-B
4.4	References	Part-III-B

Download free eBooks at bookboon.com

DEDICATION

This work is lovingly dedicated at the lotus feet of

Bhagavan Sri Sathya Sai Baba



"There is only one religion, the religion of Love.
There is only one caste, the caste of Humanity.
There is only one language, the language of the Heart.
There is only one God, He is Omnipresent."

"Help Ever, Hurt Never!"

…Bhagavan Sri Sathya Sai Baba

Download free eBooks at bookboon.com

PREFACE

This book, viz. **Cryogenic Engineering: Software solutions – Part-III-A,** is the third in the series on **Cryogenic Engineering: Software Solutions**.

This part is being published in two volumes, viz. Cryogenic Engineering: Software solutions – Part-III A and Part-III B. Part-III A contains a brief summary of background theory, definitions and formulas and the problems solved with Mathcad. Part-III B contains problems solved with Engineering Equation Solver (EES). So, it is advisable that one refers to both the parts.

As with the Part-I and II of the series, the focus is on the solutions of problems in cryogenic engineering using software such as Mathcad and Engineering Equation Solver (EES). *Only the essential theory and summary of equations required for calculations are given at the beginning of the chapter*.

Advantages of using computer software to solve problems are reiterated:

i) It helps in solving the problems fast and accurately
ii) Parametric analysis (what-if analysis) and graphical visualization is done very easily. This helps in an in-depth analysis of the problem.
iii) Once a particular type of problem is solved, it can be used as a *template* and solving similar problems later becomes extremely easy.
iv) In addition, one can plot the data, curve fit, write functions for various properties or calculations and re-use them.
v) These possibilities create interest, curiosity and wonder in the minds of students and enthuse them to know more and work more.

This book, viz. **Cryogenic Engineering: Software solutions – Part-III A** deals with the **Cryogenic refrigeration systems.**

In this book: First, thermodynamically ideal isothermal source system, concept of Coeff. of Performance (COP) and Figure of Merit (FOM) and thermodynamically ideal isobaric source system are explained.

Download free eBooks at bookboon.com

Next, Cryogenic refrigerators are dealt with under *two headings*: (i) Refrigerators for temperatures above 2 K and (ii) Refrigerators for temperatures below 2 K. The systems explained under heading (i) are: simple Linde-Hampson (L-H) refrigerator, pre-cooled L-H system, Claude refrigeration system, cold gas refrigeration system, Philips (or Stirling cycle) refrigerator, Vuilleumier cycle refrigerator, Solvay cycle and Gifford-McMahon (GM) cycle refrigerators etc. Importance of regenerators used in Philips and GM cycle refrigerators and the design equations are summarized. Under heading (ii), first, we explain: Magnetic cooling, i.e. Adiabatic demagnetization of paramagnetic materials; then, calculation of magnetic moments and entropy of paramagnetic materials are briefly outlined. Next, He^3-He^4 Dilution refrigerator is explained. Finally, Nuclear adiabatic demagnetization is mentioned.

Mathcad does not have built-in functions for properties of gases, refrigerants and fluids.

So, Mathcad functions written earlier in the free ebook, viz. "Cryogenic Engineering: Software Solutions – Vol II B" are used to get properties of cryogenic fluids.

Several Functions are written in Mathcad to simplify the standard and most required calculations. Students, teachers, researchers and professionals may find them very useful.

Many numerical problems are solved to illustrate the ease of computer calculations using Mathcad software.

S.I. Units are used throughout this book. Wide variety of worked examples presented in the book should be useful for those appearing for University, AMIE and Engineering Services examinations.

Acknowledgements: Firstly, I would like to **thank all my students**, who have been an inspiration to me in all my academic efforts.

Sincere thanks are due to **Rev. Fr. Joseph Lobo**, Director, St. Joseph Engineering College, Mangalore, for his kindness, regard and words of encouragement.

I am also thankful to **Dr. Thirumaleshwara Bhat**, Principal, Sri Madhwa Vadiraja Institute of Technology and Management, Bantakal, Udupi, for giving me support in my academic activities.

Download free eBooks at bookboon.com

I gratefully remember my former colleagues at the Cryogenics section of Technical Physics Division, Bhabha Atomic Research Centre (BARC), Bombay and Centre for Advanced Technology, Indore for their sincere cooperation in a true spirit of team work in all the projects that we undertook.

I particularly salute and admire the vision and foresight of former Heads of Technical Physics Division, BARC viz. late Mr. C. Ambasankaran, Mr. R.Y. Deshpande, Dr. S.R.Gowariker and late Mr. S.S.Ramamurthy in initiating and guiding many of the 'first of its kind' projects for Indian Space Research Organization (ISRO), wherein the Cryogenics section was deeply involved in the design and execution of the projects.

I am especially grateful to Prof. R.G. Scurlock, former Director of Institute of Cryogenics, University of Southampton, (U.K.) for writing a message for the Part I of this series.

It was indeed gracious of my former Professor, under whom I studied for M.Sc. in Cryogenics at the University of Southampton, U.K. during 1970–72, and worked as a Visiting Research Fellow during 1993–94, to honor me by writing this message.

My special thanks to **Bookboon.com** for publishing this free ebook. **Ms Karin Jakobsen** and the editorial staff have been most patient and helpful.

Finally, I would like to express my sincere thanks and appreciation to my **wife, Kala,** who, as usual, has given me continuous support, help and encouragement in all my academic activities, making many silent sacrifices.

M. Thirumaleshwar
January, 2017
Email: tmuliya@rediffmail.com

Note: Along with this book, two zip files are also available for free download. They contain several 'stand alone' .exe files to calculate performance parameters of many Cryogenics liquefiers and refrigerators. They should be of great use for students, teachers and researches for quick verification of their calculations.

Download free eBooks at bookboon.com

ABOUT THE AUTHOR

Dr. M. Thirumaleshwar graduated in Mechanical Engineering from Karnataka Regional Engineering College, Surathkal, Karnataka, India, in the year 1965. He obtained M.Sc (cryogenics) from University of Southampton, U.K. and Ph.D (cryogenics) from Indian Institute of Science, Bangalore, India.

He is a Fellow of Institution of Engineers (India), Life Member, Indian Society for Technical Education, and a Foundation Fellow of Indian Cryogenics Council.

He has worked in India and abroad on large projects in the areas involving heat transfer, fluid flow, vacuum system design, cryo-pumping etc.

He worked as Head of Cryogenics Dept. in Bhabha Atomic Research Centre (BARC), Bombay and Centre for Advanced Technology (CAT), Indore, from 1966 to 1992.

He worked as Guest Collaborator with Superconducting Super Collider Laboratory of Universities Research Association, in Dallas, USA from 1990 to 1993.

He also worked at the Institute of Cryogenics, Southampton, U.K. as a Visiting Research Fellow from 1993 to 1994.

He was Head of the Dept. of Mechanical Engineering, Fr. Conceicao Rodrigues Institute of Technology, Vashi, Navi Mumbai, India for eight years.

He also worked as Head of Dept. of Mechanical Engineering and Civil Engineering, and then as Principal, Vivekananda College of Engineering and Technology, Puttur (D.K.), India.

He was Professor and coordinator of post-graduate program in the Dept. of Mechanical Engineering in St. Joseph Engineering College, Vamanjoor, Mangalore, India.

A book entitled **"Fundamentals of Heat and Mass Transfer"** authored by him and published by M/s Pearson Education, India (2006) **has been adopted as a Text book** for third year engineering students by the Visweswaraya Technological University (V.T.U.), Belgaum, India.

Download free eBooks at bookboon.com

He has authored a *free e-book* entitled **"Software Solutions to Problems on Heat Transfer"** wherein problems are solved using 4 software viz. Mathcad, EES, FEHT and EXCEL. This book, containing about 2750 pages, is presented in 9 parts and all the 9 parts can be downloaded *for free* from www.bookboon.com

He has also authored *free e-books* on **Thermodynamics** entitled **"Basic Thermodynamics: Software Solutions" and "Applied Thermodynamics: Software Solutions"** wherein problems are solved using 3 software viz. Mathcad, EES, and TEST. Each of these titles is presented in 5 parts and all the books can be downloaded *for free* from www.bookboon.com

In addition, he has authored following two useful *free ebooks*:

i) Applied Thermodynamics: Software Solutions: Vapor compression Refrigeration cycle + Problems (Mathcad)

http://bookboon.com/en/applied-thermodynamics-software-solutions-vapor-ebook

ii) Applied Thermodynamics: Software Solutions: Vapor Power cycles (Rankine cycle) + Problems (Mathcad)

http://bookboon.com/en/applied-thermodynamics-software-solutions-vapor-po-ebook

His earlier *free ebooks in this* **Cryogenic Engineering series**, viz. **Cryogenic Engineering: Software Solutions – Parts I, II-A and II-B** were published by Bookboon about an year ago.

He has also authored **three motivational, free ebooks**, published by www.bookboon.com, entitled as follows:

1. Towards Excellence… How to Study (A Guide book to Students)
2. Towards Excellence… How to teach (A guide book to Teachers)
3. Towards Excellence… Seminars, GD's and Personal Interviews

Dr. M. Thirumaleshwar has attended several National and International conferences and has more than 50 publications to his credit.

Download free eBooks at bookboon.com

ABOUT THE SOFTWARE USED

Following three software are used while solving problems in this book series:

1. Mathcad 7 and Mathcad 15 (Ref: www.ptc.com)
2. Engineering Equation Solver (EES) (Ref: www.fchart.com), and

For a brief introduction to Mathcad, EES and EXCEL see the chapter 1 of the following *free ebook* by the author:

"Software Solutions to Problems on Heat Transfer – CONDUCTION Part-I":
http://bookboon.com/en/software-solutions-to-problems-on-heat-transfer-ebook

be > your degree
© 2013 Accenture. All rights reserved.
Bring your talent and passion to a global organization at the forefront of business, technology and innovation. Discover how great you can be.
Visit accenture.com/bookboon
Be greater than.
consulting | technology | outsourcing
accenture
High performance. Delivered.
Download free eBooks at bookboon.com
Click on the ad to read more

4 CRYOGENIC REFRIGERATION SYSTEMS

Learning objectives:

1. In this chapter, topics on 'cryogenic refrigeration systems' are dealt with.
2. First, thermodynamically ideal isothermal source system, concept of Coeff. of Performance (COP) and Figure of Merit (FOM) are explained. Thermodynamically ideal isobaric source system is also explained.
3. Next, cryogenic refrigerators are explained under *two headings*: (i) Refrigerators for temperatures above 2 K and (ii) refrigerators for temperatures below 2 K.
4. The systems dealt with under heading (i) are: simple Linde-Hampson (L-H) refrigerator, pre-cooled L-H system, Claude refrigeration system, cold gas refrigeration system, Philips (or Stirling cycle) refrigerator, Vuilleumier cycle refrigerator, Solvay cycle and Gifford-McMahon (GM) cycle refrigerators etc. Importance of regenerators used in Philips and GM cycle refrigerators and the design equations are summarized.
5. Under heading (ii), first, we explain: Magnetic cooling, i.e. Adiabatic demagnetization of paramagnetic materials; then, calculation of magnetic moments and entropy of paramagnetic materials are briefly outlined. Next, He^3-He^4 Dilution refrigerator is explained. Finally, Nuclear adiabatic demagnetization is also mentioned.
6. Many numerical problems are solved to illustrate the ease of computer calculations using Mathcad software.

==========

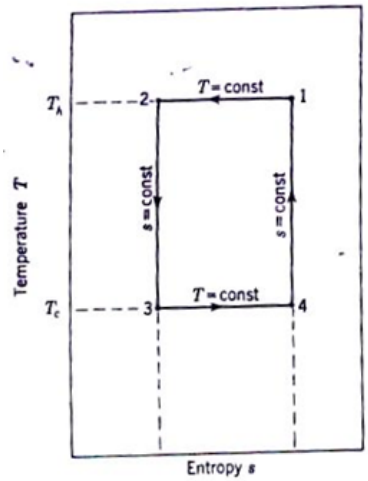
Download free eBooks at bookboon.com

4.1 DEFINITIONS, STATEMENTS AND FORMULAS USED[1-9]:

4.1.1 THERMODYNAMICALLY IDEAL, ISOTHERMAL SOURCE SYSTEM [1]:

Carnot cycle is the Ideal cycle.

T-s diagram is shown below:



Here, refrigeration is supplied at T3 (= T4) and heat is rejected at T2 (= T1).

Net work is equal to the area of the cycle T-s diagram = area 1-2-3-4

Refrigeration = area under the line 3-4

Coeff. of Performance (COP) for Ideal cycle = Refrigeration/Net work.

We get:

$$COP_i = \frac{T_c}{T_h - T_c}$$

COP of Ideal cycle depends only on temperatures (in Kelvin) and not on the refrigerant substance.

Download free eBooks at bookboon.com

COP and the Work requirement per unit refrigeration for various source temps are given below (for a sink temp of 300 K)[1]:

Source Temperature			
K	°R	$COP_i = -Q_s/W_{net}$	$-W_{net}/Q_s$
111.7	201.1	0.5932	1.686
77.4	139.3	0.3477	2.876
20.3	36.5	0.07258	13.778
4.2	7.56	0.01420	70.43
1.0	1.8	0.003344	299.0
0.1	0.18	0.0003334	2,999.0
0.01	0.018	0.0000333	29,999.0

Note that as the source temp (T_c) decreases, COP decreases and work requirement increases drastically.

The Wake
the only emission we want to leave behind
Low-speed Engines Medium-speed Engines Turbochargers Propellers Propulsion Packages PrimeServ
The design of eco-friendly marine power and propulsion solutions is crucial for MAN Diesel & Turbo.
Power competencies are offered with the world's largest engine programme – having outputs spanning
from 450 to 87,220 kW per engine. Get up front!
Find out more at www.mandieselturbo.com
Engineering the Future – since 1758.
MAN Diesel & Turbo
MAN

Download free eBooks at bookboon.com
Click on the ad to read more

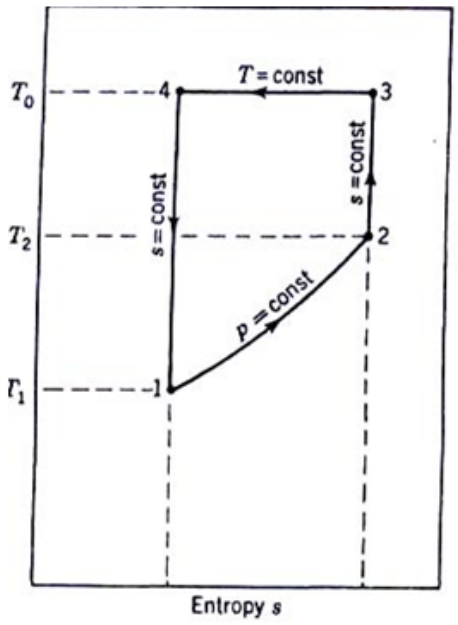
For a 'real system', we define a Figure of Merit (FOM) to compare with the Ideal system:

$FOM = COP/COP_i$

where COP is for the real system and COP_i is for the Ideal system.

4.1.2 THERMODYNAMICALLY IDEAL, ISOBARIC SOURCE SYSTEM [1]:

Here, refrigeration occurs at constant pressure, and at varying temperatures:
T-s diagram is shown below:



We have:

$$Q_a = \int_1^2 mT\ ds = m\int_1^2 (dh - \upsilon dp) = m(h_2 - h_1)$$

$$W_{net} = Q_{net} = Q_r + Q_a = -m\left[T_0(s_2 - s_1) - (h_2 - h_1)\right]$$

(-ve sign only indicates that work is supplied to the system.)

And, COP:

$$COP_i = \frac{-Q_a}{W_{net}} = \frac{h_2 - h_1}{T_0(s_2 - s_1) - (h_2 - h_1)}$$

Download free eBooks at bookboon.com

And, for an Ideal gas:

$$h_2 - h_1 = c_p\left(T_2 - T_1\right)$$

$$s_2 - s_1 = c_p \ln\left(T_2 / T_1\right) - R\ \ln\left(p_2 / p_1\right) = c_p \ln\left(T_2 / T_1\right)$$

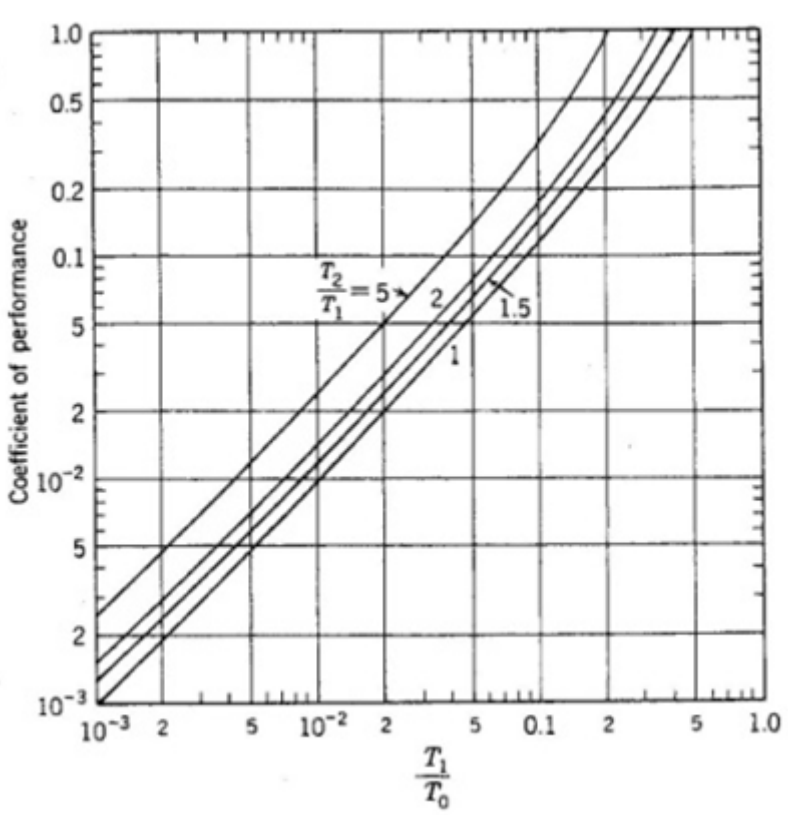
And,

$$COP_i = \frac{T_2 - T_1}{T_0 \ln\left(T_2 / T_1\right) - \left(T_2 - T_1\right)}$$

Or,

$$COP_i = \frac{\left(T_2 / T_1\right) - 1}{\left(T_0 / T_1\right)\ln\left(T_2 / T_1\right) - \left(T_2 / T_1\right) + 1}$$

Plot of COP vs temp ratio:

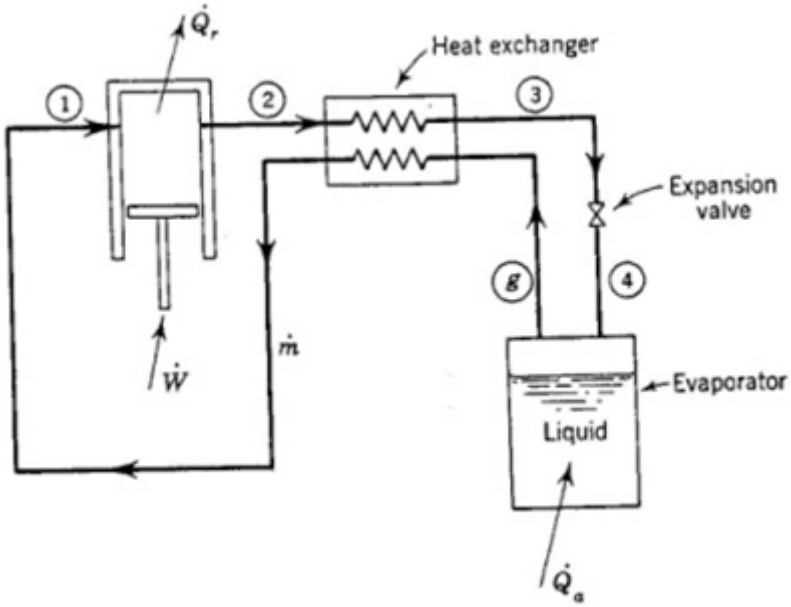


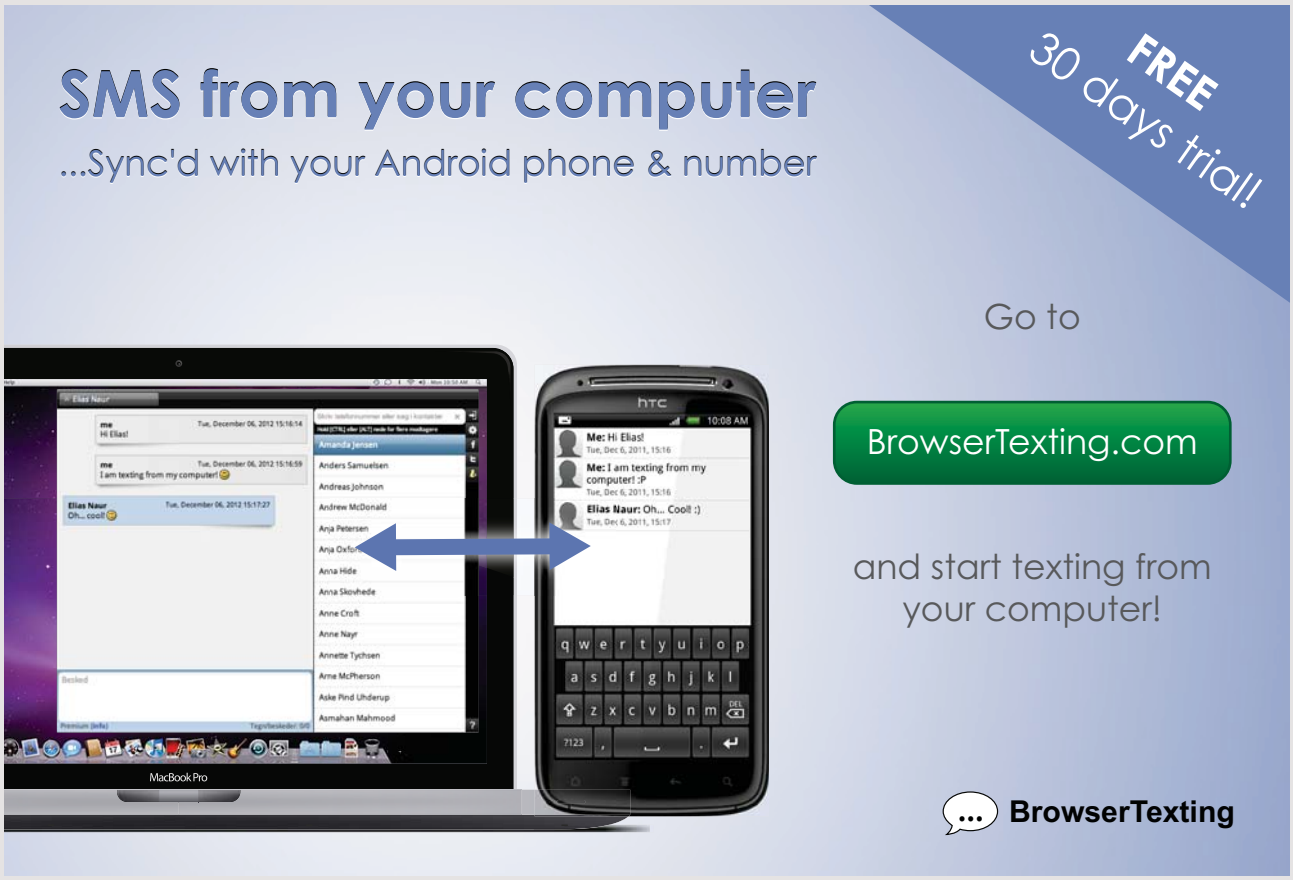
Download free eBooks at bookboon.com

4.1.3 JOULE-THOMSON (J-T) REFRIGERATION SYSTEMS [1]:

Linde-Hampson (L-H) refrigerator:

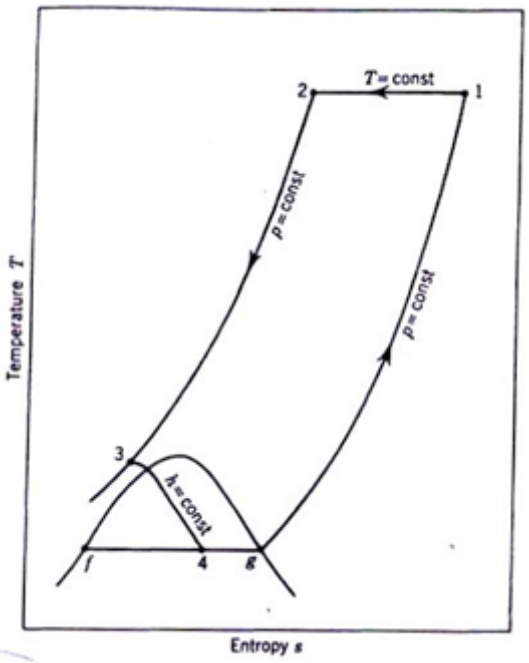
The schematic diagram and the T-s diagram are shown below:





Download free eBooks at bookboon.com

Click on the ad to read more



Applying the First Law of Thermodynamics to the system consisting of the heat exchanger, expansion valve, and evaporator and assume no heat in-leaks from ambient as well as negligible kinetic-energy and potential-energy changes of the working fluid, we obtain

$$\dot{Q}_a = \dot{m}\left(h_1' - h_2\right)$$

where h_1' is the actual enthalpy of the fluid leaving the heat exchanger at the warm end. The heat exchanger effectiveness is defined by

$$\in = \frac{h_1' - h_g}{h_1 - h_g}$$

And:

$$\dot{Q}_a / \dot{m} = \left(h_1 - h_2\right) - \left(1 - \in\right)\left(h_1 - h_g\right)$$

Download free eBooks at bookboon.com

Note *two* points: First, Joule-Thomson refrigerator cannot be used with neon, hydrogen, or helium as the working medium, unless these gases are first pre-cooled below their maximum inversion temperatures. Recollect **[1, 8]**:

Gas	Max. Inversion temp (K)
Helium-4	45
Hydrogen	205.
Neon	250
Nitrogen	621
Air	603
Carbon monoxide	652
Argon	794
Oxygen	761
Methane	939
Carbon dioxide	1500
Ammonia	1994

Second, we see that there is a value of the heat exchanger effectiveness below which the refrigerator will not work. This limiting effectiveness may be determined by setting the refrigeration effect equal to zero in above eqn.

The work requirement for the system is given by

$$\frac{-\dot{W}}{m} = \frac{T_2(s1-s2)-(h1-h2)}{\eta_{co}}$$

where η_{co} is the overall efficiency of the compressor.

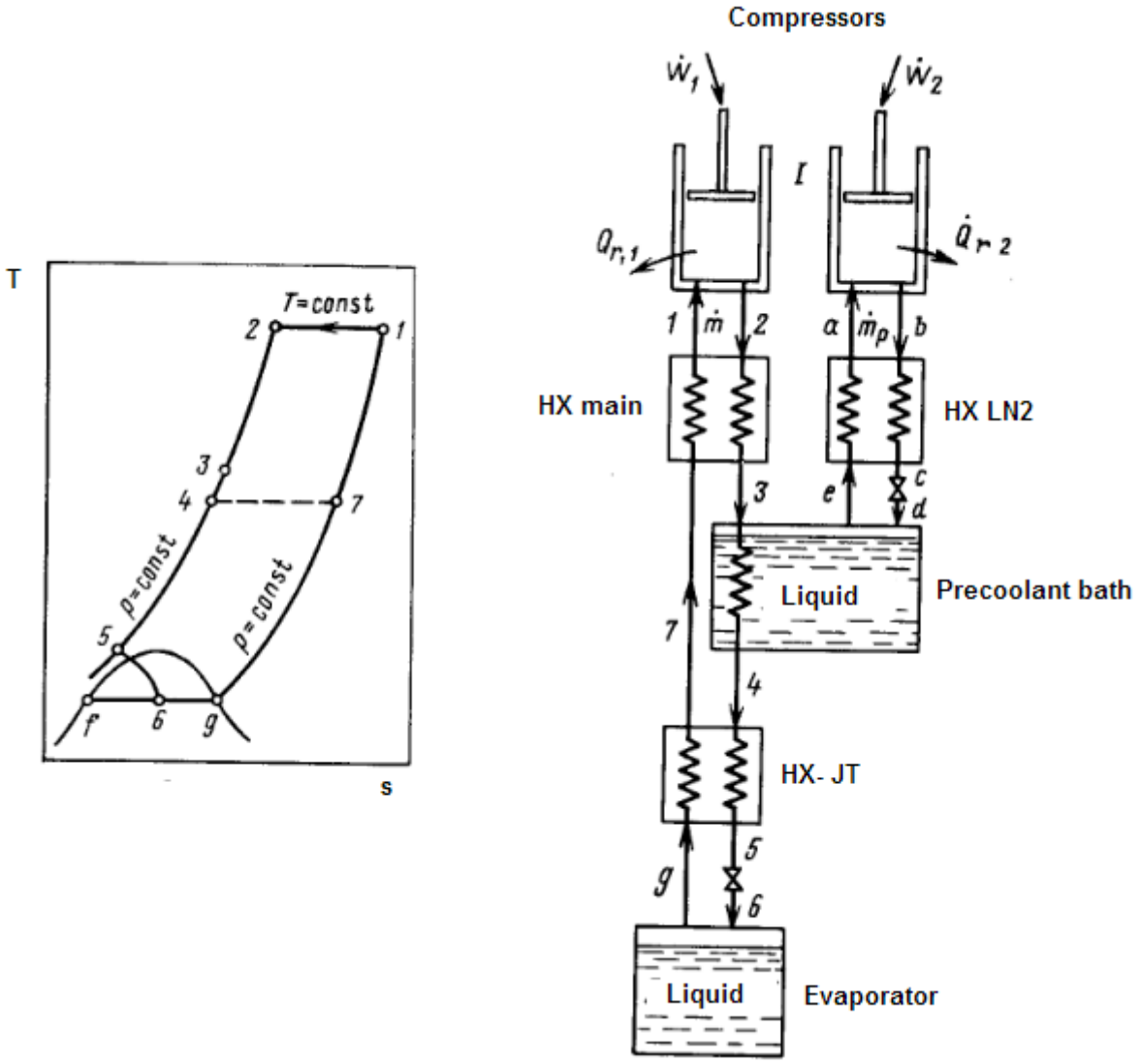
Then, we find the COP for the Linde-Hampson refrigerator to be

$$\text{COP} = \frac{-\dot{Q}_a}{\dot{W}} = \frac{\eta_{c,0}\left[\left(h_1-h_2\right)-\left(1-\in\right)\left(h_1-h_g\right)\right]}{T_2\left(s_1-s_2\right)-\left(h_1-h_2\right)}$$

Download free eBooks at bookboon.com

4.1.4 PRE-COOLED LINDE-HAMPSON (L-H) REFRIGERATOR [1]:

Typical diagrams for a H2 refrigeration system with LN2 pre-cooling is shown below:



Using subscript 'p' to indicate pre-coolant:

Applying the First Law of Thermodynamics to all components of the system shown in Fig. 4.7 except the compressors, neglect heat inleaks from ambient, and neglect kinetic-energy and potential-energy changes of the fluids, we obtain

$$\dot{Q}_a = \dot{m}\left(h_1' - h_2\right) + \dot{m}_p\left(h_a' - h_b\right)$$

Download free eBooks at bookboon.com

Defining the pre-coolant mass-flow-rate ratio as

$$z = \dot{m}_p / \dot{m}$$

we get:

$$\dot{Q}_a / \dot{m} = (h_1' - h_2) + z(h_a' - h_b)$$

Introducing the effectivenesses of main heat exchanger and pre-coolant heat exchangers:

$$\in = \frac{h_1' - h_g}{h_1 - h_g}$$

$$\in_p = \frac{h_a' - h_e}{h_a - h_e}$$

Making these substitutions, we get:

$$\dot{Q}_a / \dot{m} = (h_1 - h_2) - (1 - \in)(h_1 - h_g)$$

$$+ z\left[(h_a - h_b) - (1 - \in_p)(h_a - h_e)\right] \quad \ldots \text{(eqn. A)}$$

Download free eBooks at bookboon.com
Click on the ad to read more

where h_1 is the enthalpy of the main refrigerant at the temperature T_2, and h_a is the enthalpy of the pre-coolant at the temperature T_b.

Applying the First Law to the cold exchanger and the evaporator, we obtain:

$$Q_a/m = h'_7 - h_4$$

Introducing the effectiveness of the cold exchanger,

$$\epsilon_c = \frac{h'_7 - h_g}{h_7 - h_g}$$

And, the refrigeration effect:

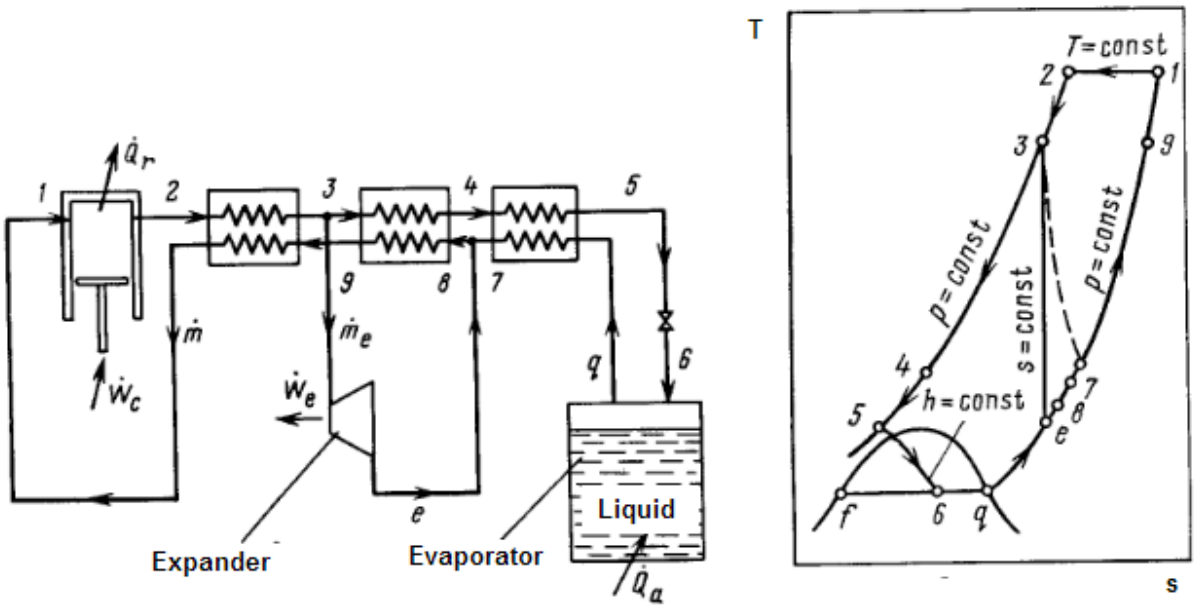
$$\dot{Q}_a / \dot{m} = (h_7 - h_4) - (1 - \epsilon_c)(h_7 - h_g) \quad \ldots \text{(eqn. B)}$$

The required pre-coolant mass-flow-rate ratio may be determined by equating eqns. (A) and (B), if we assume that the temperature at point 4 is practically equal to the pre-coolant bath temperature so that h_4 and h_7 are known quantities.

4.1.5 EXPANSION ENGINE REFRIGERATION SYSTEMS [1]:

Claude refrigeration system:

Schematic and T-s diagrams are shown below:



Download free eBooks at bookboon.com

Applying the First Law to the three heat exchangers, the expansion valve, and the evaporator as a unit, neglecting heat in-leaks from ambient and kinetic-energy and potential-energy changes, we get for the heat absorbed by the refrigerant:

$$\dot{Q}_a / \dot{m} = (h_1' - h_2) + x(h_3 - h_e')$$

where $x = \dot{m}_e / \dot{m}$ = expander mass-flow-rate ratio, $\dot{m}_e$ = mass flow rate through the expander, $\dot{m}$ = mass flow rate through the compressor, and the subscripts refer to the points given in Fig. above.

The expression for the refrigeration effect may be written in terms of the expander adiabatic efficiency as follows:

$$\dot{Q}_a / \dot{m} = (h_1' - h_2) + x\eta_{ad}(h_3 - h_e)$$

Brain power
By 2020, wind could provide one-tenth of our planet's electricity needs. Already today, SKF's innovative know-how is crucial to running a large proportion of the world's wind turbines.
Up to 25 % of the generating costs relate to maintenance. These can be reduced dramatically thanks to our systems for on-line condition monitoring and automatic lubrication. We help make it more economical to create cleaner, cheaper energy out of thin air.
By sharing our experience, expertise, and creativity, industries can boost performance beyond expectations.
Therefore we need the best employees who can meet this challenge!
The Power of Knowledge Engineering
Plug into The Power of Knowledge Engineering.
Visit us at www.skf.com/knowledge
SKF
Download free eBooks at bookboon.com
Click on the ad to read more

The net work requirement, assuming that the expander work is utilized to help in the compression of the gas, is given by

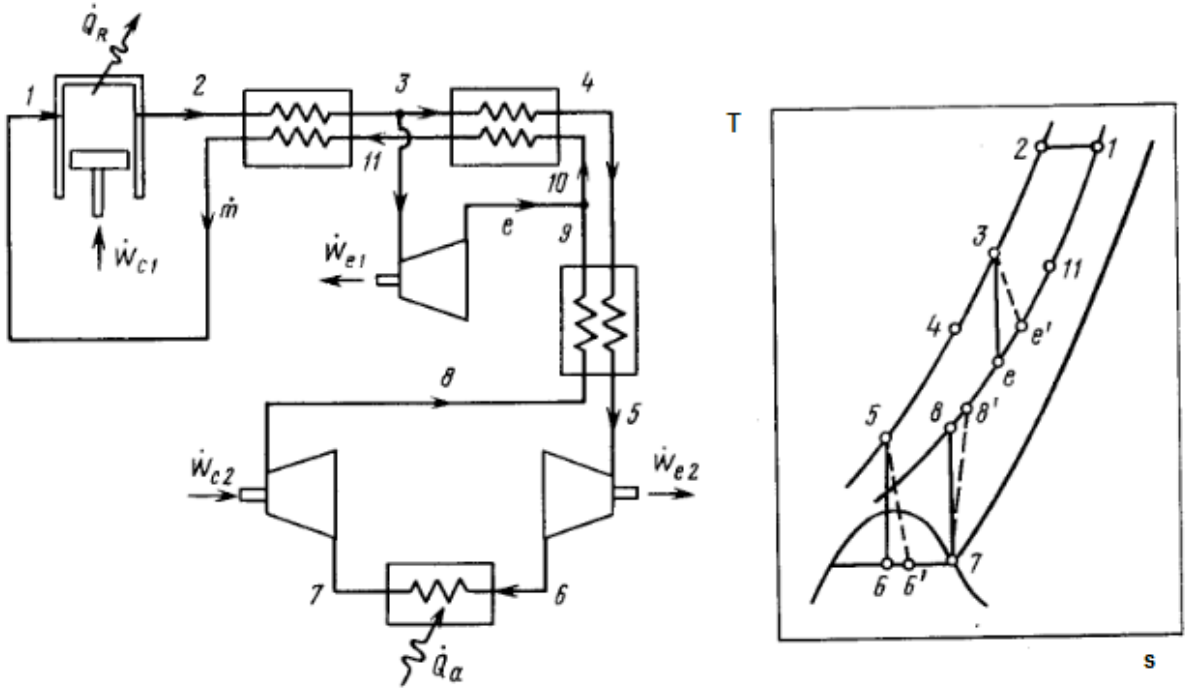
$$-\dot{W}_{net}/\dot{m} = \left[T_2(s_1 - s_2) - (h_1 - h_2)\right]/\eta_{c,0} - x\eta_{e,m}\eta_{ad}(h_3 - h_e)$$

where $h_{c,0}$ is the overall efficiency of the compressor and $h_{e,m}$ is the mechanical efficiency of the expander.

Two significant modifications of the basic Claude system are the use of a "wet" expander or expander operating in the two-phase region to replace the expansion valve, and the use of a low temperature compressor.

A schematic of this system and the T-s diagram are shown below.

The two-phase expander is used primarily for systems involving helium as the working fluid because the thermal capacity of the compressed gas is generally larger than the latent heat of the liquid phase. Unlike an air or nitrogen expansion engine in which the engine efficiency is seriously affected by the presence of liquid in the engine, operation of the helium expander in the two-phase region does not result in a serious deterioration of the engine performance. The thermodynamic performance of the system is improved by the use of the saturated-vapor compressor. In addition, the required heat exchanger surface is less than that required for the conventional Claude system because of heat-transfer coefficients are higher when the cold gas stream pressure is increased.



Download free eBooks at bookboon.com

4.1.6 PRACTICAL LARGE SCALE HELIUM REFRIGERATORS [10, 11]:

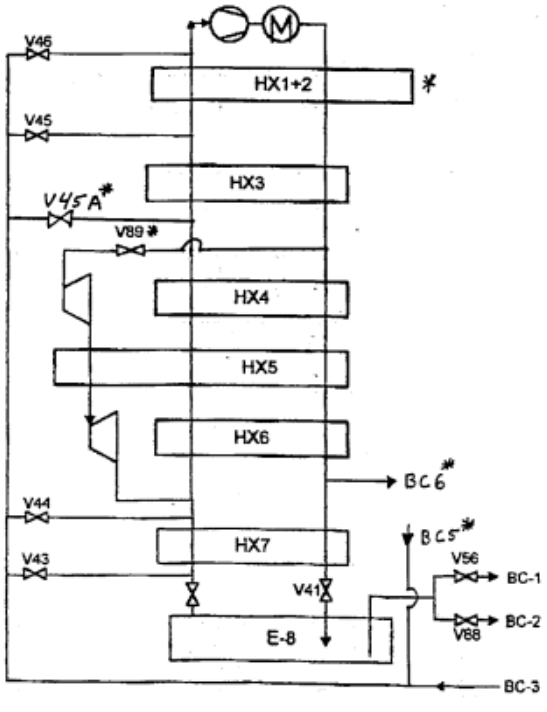
Modern large scale Helium refrigerators/liquefiers use a variation of the Claude cycle known as the **Collins cycle**. Claude cycle uses one expansion engine, whereas Collins cycle uses multiple expansion engines. The post WW II development of the Collins liquefier revolutionized laboratory research in cryogenics.

Collins cycle consists of:

1) Compression to ~ 16 Bar with cooling back to 300 K + oil removal
ii) Cooling of high pressure gas with LN2
iii) Isentropic expansion via 2 or more expansion engines
iv) Cooling of high pressure gas by the cold returning low pressure stream
v) Isenthalpic expansion through JT valve
vi) Return of gas to compressors at just above 1 Bar

Two examples of large Collins refrigerators:

1. **CTI 4000 refrigerator (1.2 kW at 4.5 K – early 1980s):**



CTI 4000 Upgrade 12 / 2 / 99

* Indicates new or changed component

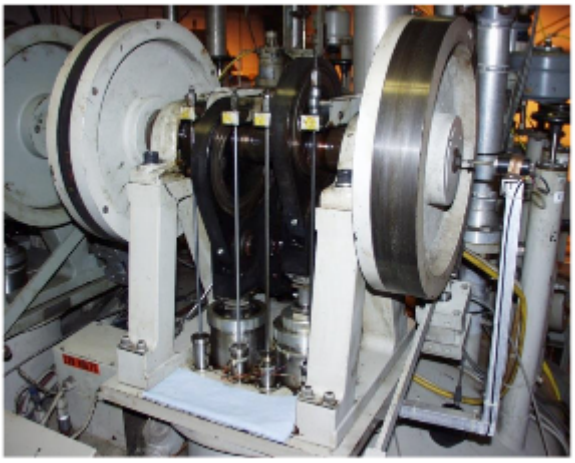
Download free eBooks at bookboon.com

Photo is shown below:

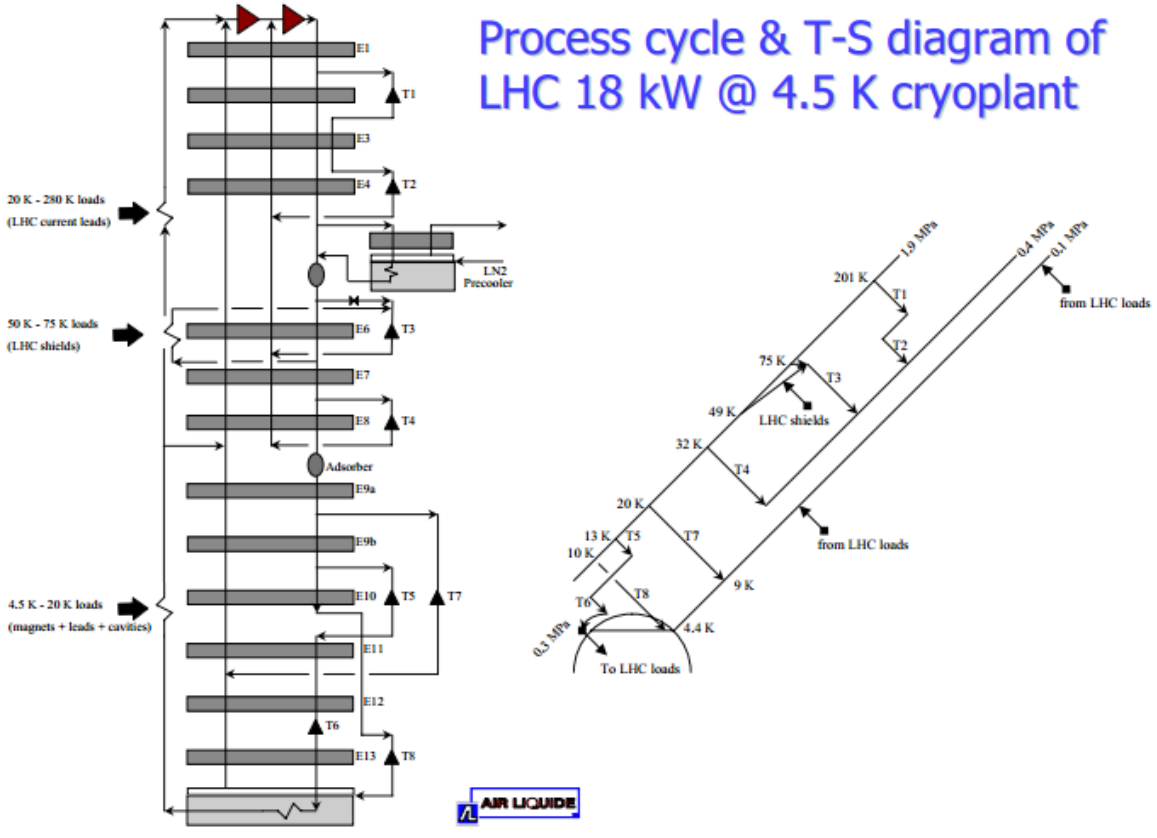
Download free eBooks at bookboon.com

Click on the ad to read more

Reciprocating expansion engines: 2 nos., rated at 250 rpm. Each has two cylinders, 9.1 cm bore, 7.6 cm stroke (largest in the world!), photo is shown below:

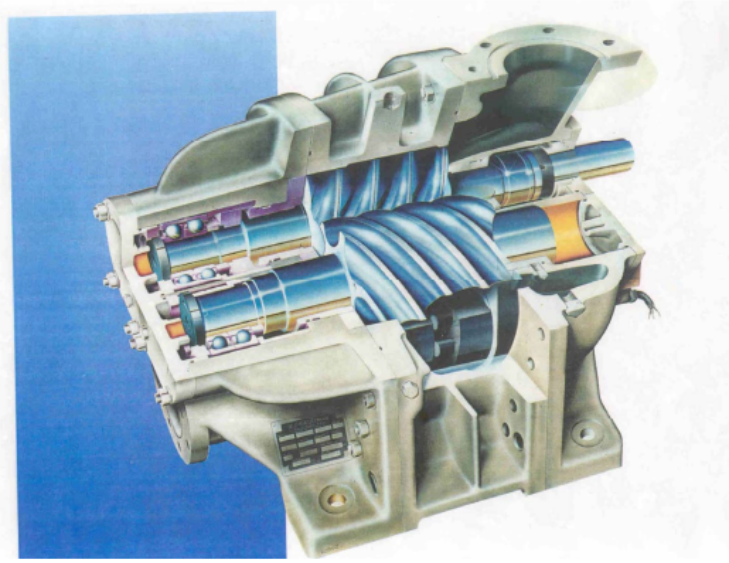


2. **LHC 4.5 K refrigeration plant (18 kW at 4.5 K – produced in 2004; totally 8 nos. required; 4 from Linde, 4 from Air Liquide):**



Download free eBooks at bookboon.com

Oil-injected screw compressor



LHC 18 kW @ 4.5 K helium cryoplants

33 kW @ 50 K to 75 K
23 kW @ 4.6 K to 20 K
41 g/s liquefaction
4 MW compressor power
C.O.P. 220-230 W/W @ 4.5 K

Air Liquide

Linde

Download free eBooks at bookboon.com

Compressor station of LHC 18 kW@ 4.5 K helium refrigerator

LIGS University

based in Hawaii, USA

is currently enrolling in the Interactive Online BBA, MBA, MSc, DBA and PhD programs:

- enroll by October 31st, 2014 and
- save up to 11% on the tuition!
- pay in 10 installments / 2 years
- Interactive Online education
- visit www.ligsuniversity.com to find out more!

Note: LIGS University is not accredited by any nationally recognized accrediting agency listed by the US Secretary of Education.
More info here.

Download free eBooks at bookboon.com

Click on the ad to read more

Cold compressors for 2.4 kW @ 1.8 K refrigeration units

(a)

Air Liquide

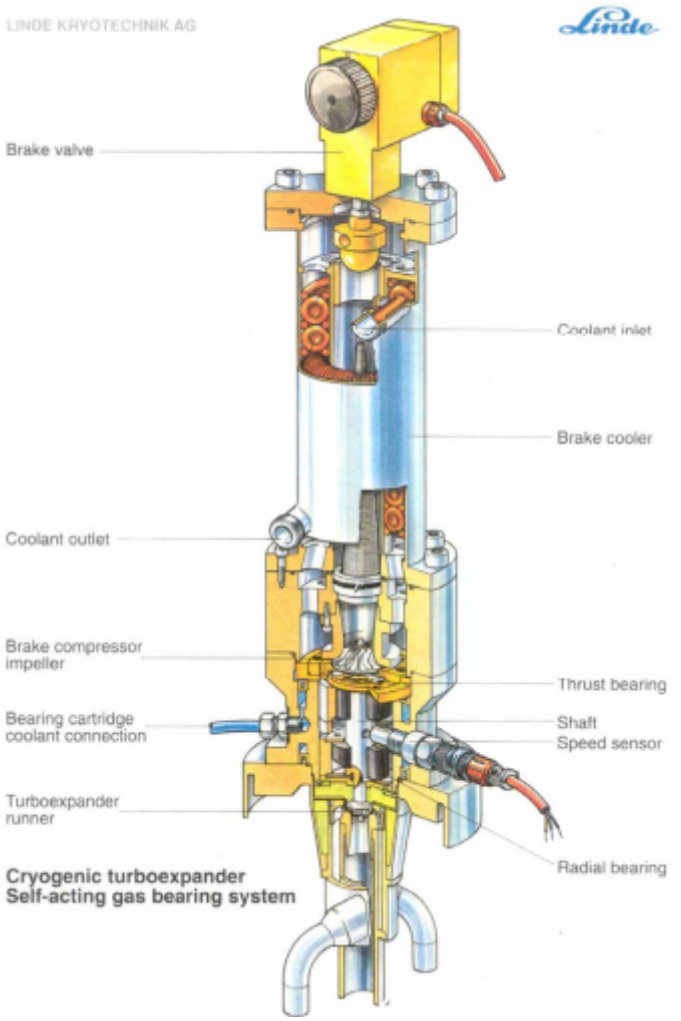
(b)

IHI/Linde

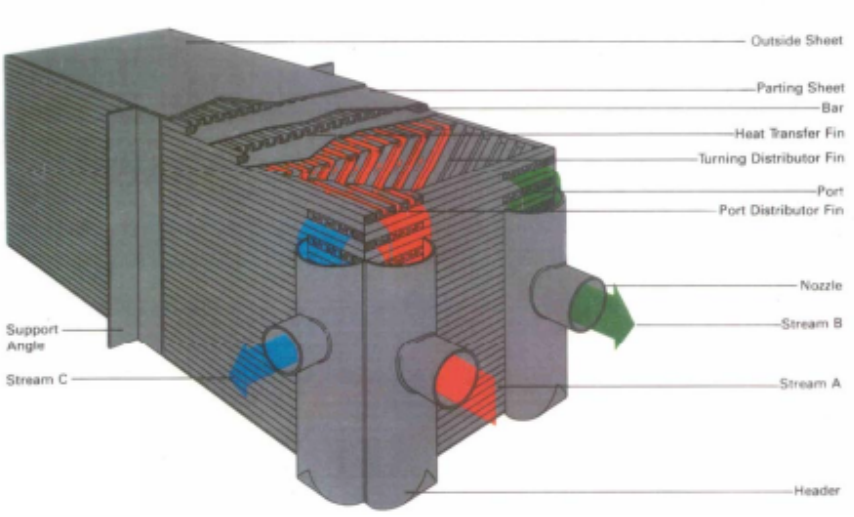
Cryogenic turbo expanders -- Air Liquide

Cryogenic turbo expander --- Linde

Download free eBooks at bookboon.com



Brazed Aluminium plare fin heat exchanger



Download free eBooks at bookboon.com

Note: These machines have:

i) Large number of expansion turbines (speeds approx. 130,000 rpm) – some in series with HP stream
ii) Medium pressure return
iii) Heat loads at intermediate temperatures
iv) Designed to have high % Carnot (roughly 30%)

Major components of a helium refrigeration plant:

i) **Helium compressors:** operating at room temp. water cooled, oil flooded, large power required (up to 2000 HP)
ii) **Oil removal systems:** removes oil up to ppb level, contains activated charcoal which can be regenerated with warm N2 gas
iii) **Cold box:** cold box is vacuum insulated, containing heat exchangers (brazed plate – fin type), valves, expansion turbines, piping.

TURN TO THE EXPERTS FOR
SUBSCRIPTION CONSULTANCY

Subscrybe is one of the leading companies in Europe when it comes to innovation and business development within subscription businesses.

We innovate new subscription business models or improve existing ones. We do business reviews of existing subscription businesses and we develope acquisition and retention strategies.

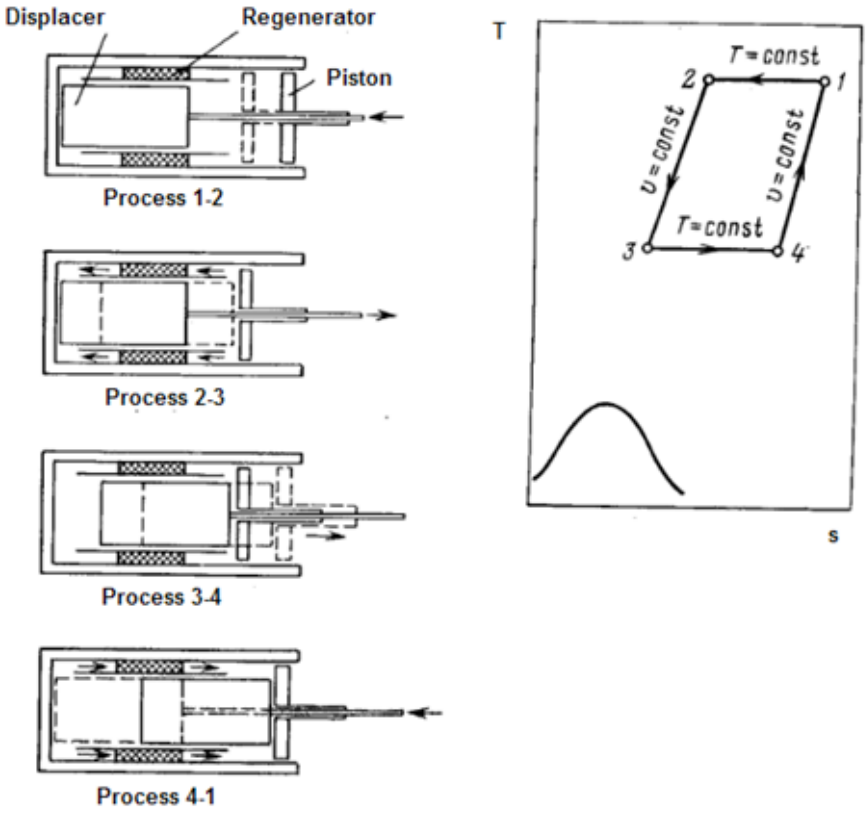
Learn more at linkedin.com/company/subscrybe or contact
Managing Director Morten Suhr Hansen at mha@subscrybe.dk

SUBSCRYBE - to the future

Download free eBooks at bookboon.com
Click on the ad to read more

4.1.7 PHILIPS REFRIGERATOR [1, 18]:

Schematic diagram and T-s diagram are shown below:



The sequence of operations for the system is as follows.

Process 1-2: The gas is compressed isothermally while rejecting heat to the high-temperature sink (surroundings).

Process 2-3: The gas is forced through the regenerator by the motion of the displacer. The gas is cooled at constant volume during this process. The energy removed from the gas is not transferred to the surroundings but is stored in the regenerator matrix.

Process 3-4: The gas is expanded isothermally while absorbing heat from the low-temperature source.

Process 4-1: The cold gas is forced through the regenerator by the motion of the displacer; the gas is heated during this process. The energy stored during process 2–3 is transferred back to the gas. In the ideal case (no heat in-leaks), heat is transferred to the refrigerator only during process 3–4, and heat is rejected from the refrigerator only during process 1–2.

Download free eBooks at bookboon.com

We have:

Heat rejected = $Q_r = mT_1(S_2 - s_1)$

Heat absorbed = $Q_a = mT_3(s_4 - s_3)$

And,

$$COP = \frac{-Q_a}{W_{net}} = \frac{T_3}{T_1(s_1 - s_2)/(s_4 - s_3) - T_3}$$

If the working fluid behaves as an ideal gas, we may write

$$s_1 - s_2 = c_\upsilon \ln(T_1/T_2) + R\ln(\upsilon_1/\upsilon_2)$$

$$= R\ln(\upsilon_1/\upsilon_2) = R\ln(\upsilon_4/\upsilon_3) = s_4 - s_3$$

because $T_1 = T_2$ and $T_3 = T_4$, $\upsilon_1 = \upsilon_4$ and $\upsilon_2 = \upsilon_3$, where υ is the gas specific volume. The coefficient of performance of an ideal Philips refrigerator with an ideal gas as the refrigerant is

$$COP = \frac{T_3}{T_1 - T_3}$$

This is the same expression as that for the COP of a Carnot refrigerator; therefore, the ideal Philips Refrigerator would have a FOM of unity.

4.1.8 TWO EXAMPLES OF COMMERCIAL STIRLING CYCLE CRYOCOOLERS:

A) **From brochure of M/s Sunpower Inc. OH, USA:**
(Ref: http://sunpowerinc.com/cryocoolers/)

These are used in the medical, instrumentation, security, astronomy, telecommunications, and pharmaceuticals markets.

Download free eBooks at bookboon.com

Summary specifications:

	Split Series		Integral Series		
Generation II Controller	DS 1.5	DS 2.1*	MT	CT	GT
Nominal Lift @77k (23°C reject)	1.5 W	2.1 W	5 W	11 W	16 W
Nominal Cooler Input Power	30 W	43 W	80 W	160 W	240 W
Cooler Mass	1.2 kg	1.3 kg	2.1 kg	3.1 kg	3.1kg
Overall Length	Cold Head: 128 mm Wave Generator: 116 mm	Cold Head: 128 mm Wave Generator: 116 mm	242.8 mm	260.0 mm	275.5 mm
Outside Diameter	Cold Head: 39 mm Wave Generator: 55 mm	Cold Head: 39 mm Wave Generator: 55 mm	73.0 mm	83 mm	83 mn
MTTF	120,000 hrs Goal	120,000 hrs Goal	200,000 hrs	200,000 hrs	200,00 hrs

Download free eBooks at bookboon.com

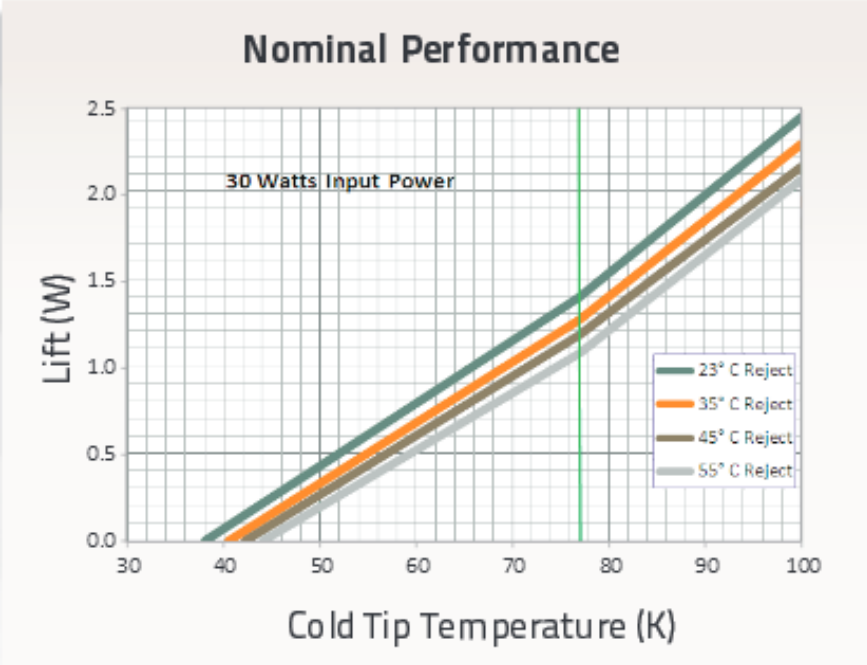
Two typical examples:

1. **Model CryoTel DS 1.5 (1,4 W Cryocooler): Nominal Lift at 77 K = 1.4W**

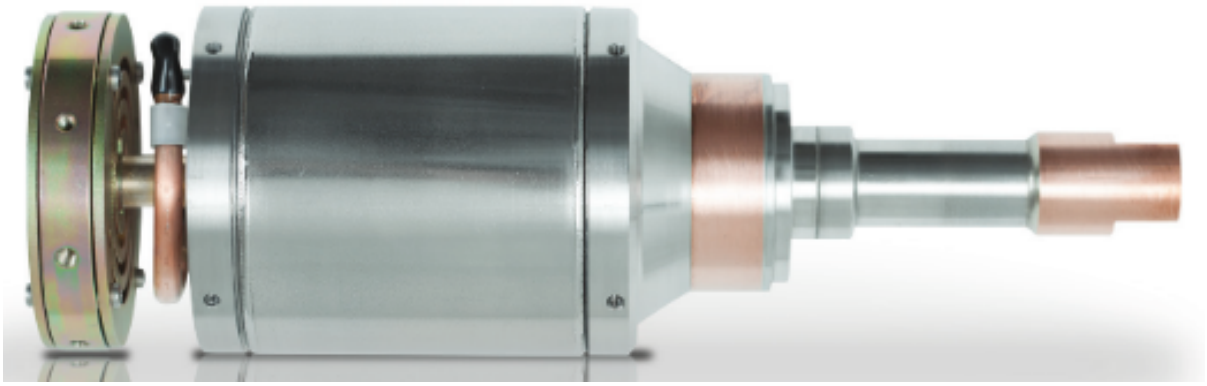
"I studied English for 16 years but... ...I finally learned to speak it in just six lessons"
Jane, Chinese architect
ENGLISH OUT THERE
Click to hear me talking before and after my unique course download

Download free eBooks at bookboon.com

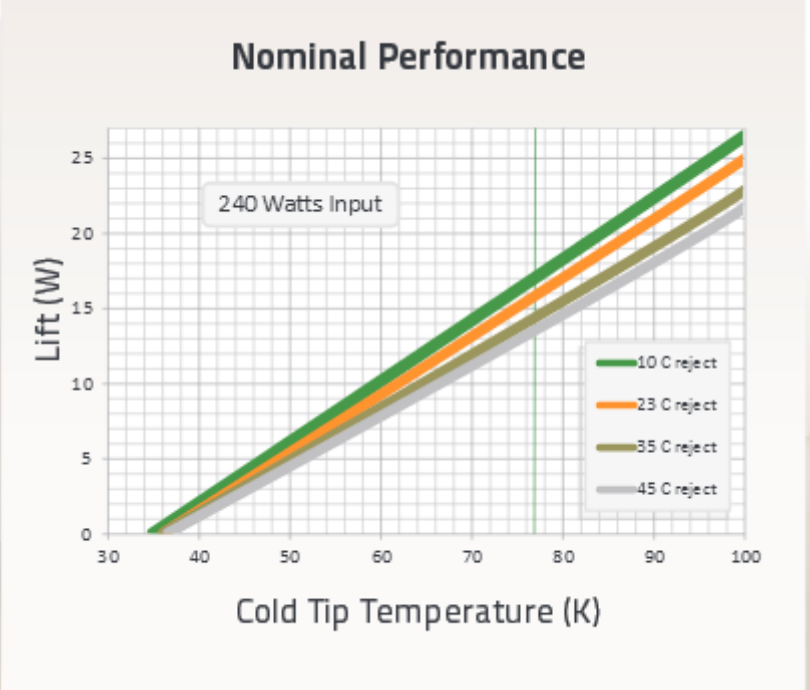
Click on the ad to read more

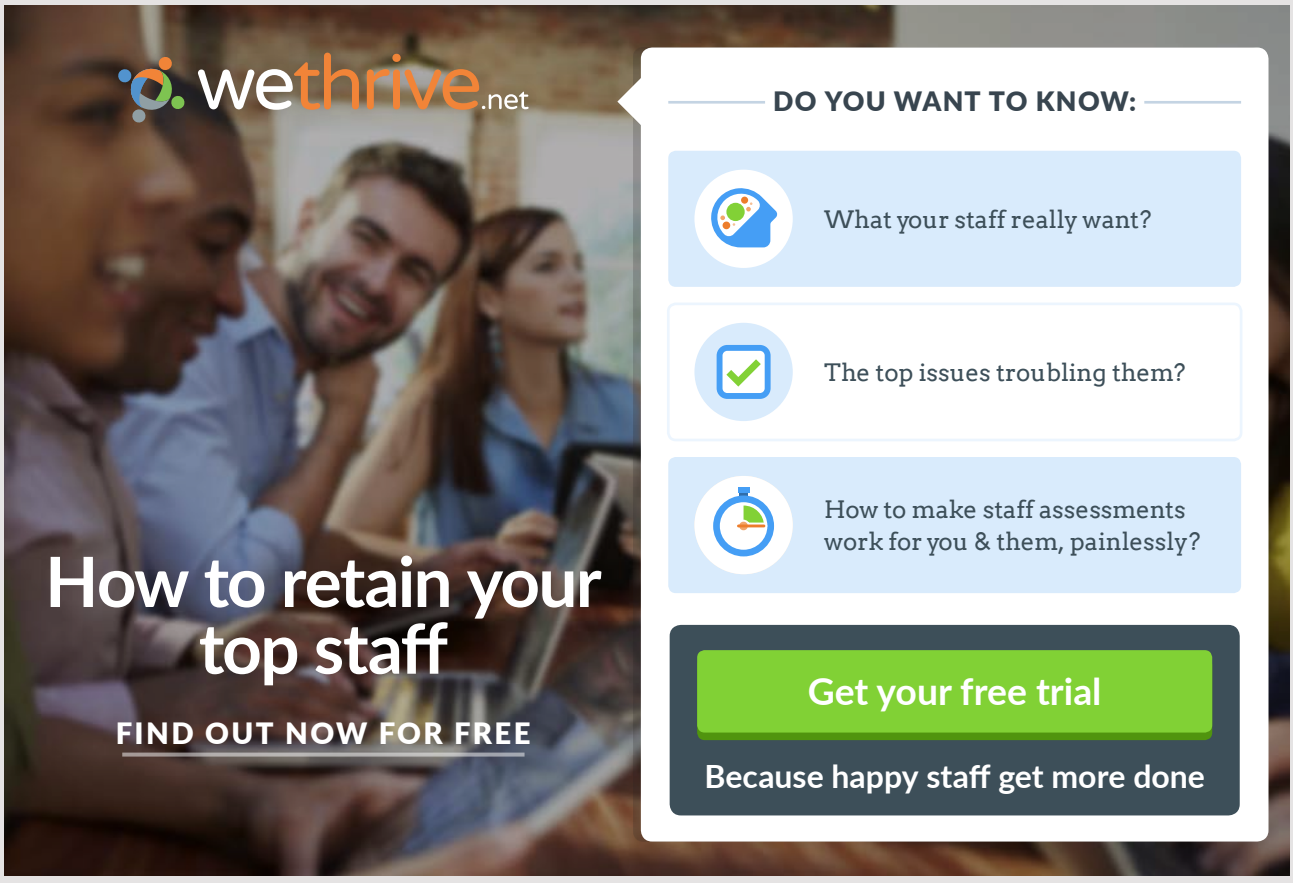


2. **Model CryoTel GT (16 W Cryocooler): Nominal Lift at 77 K = 16 W.**



Download free eBooks at bookboon.com




wethrive.net
How to retain your top staff
FIND OUT NOW FOR FREE
DO YOU WANT TO KNOW:
What your staff really want?
The top issues troubling them?
How to make staff assessments work for you & them, painlessly?
Get your free trial
Because happy staff get more done

Download free eBooks at bookboon.com
Click on the ad to read more

B. **Larger cryogenerators based on Philips Stirling cycle:**

Following information is from the website of Stirling cryogenics, India, who acquired Cryogenics business of Philips India Limited in 2002.

Ref: https://www.stirlingcryogenics.com/en/products/cryocoolers

---

The SPC-4T is a two-stage cryogenerator that provides cooling power in the range of 120–700W @ 15-60K. The SPC-4T is mostly used for (indirect) cooling of a subject by means of forced flow Helium gas.

See photo below:

SPC-4T Cryogenerator

The SPC-1T is a two-stage cryogenerator that provides cooling power in the range of 30-175W @ 15-60K. The SPC-1T is mostly used for (indirect) cooling of a subject by means of forced flow Helium gas.

Download free eBooks at bookboon.com

See photo below:

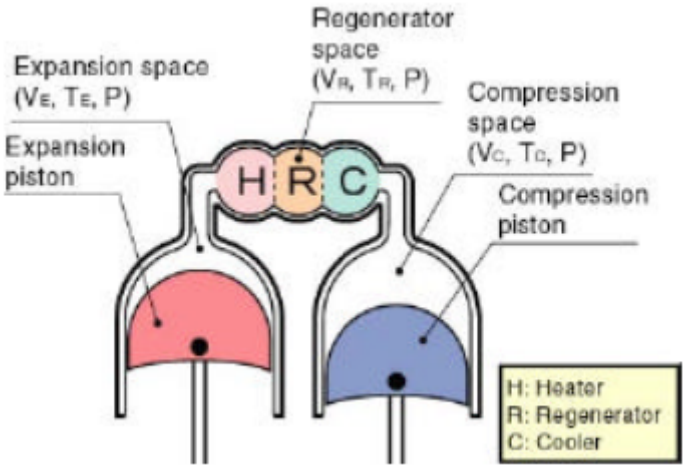
4.1.9 SCHMIDT'S THEORY FOR ANALYSIS OF STIRLING CYCLE [19]:

In the ideal Stirling cycle, the heat supply and rejection occur isothermally, and the motion of displacer and piston are intermittent. However, in an actual, practical Stirling cycle, the motion of displacer and piston are sinusoidal; the heat transfer processes are also *not* isothermal.

Schmidt's analysis assumes sinusoidal motion for displacer and piston; however, the heat transfers are still assumed to be isothermal. Even then, this theory is nearer to the actual Stirling cycle, and is a good first step in the design process.

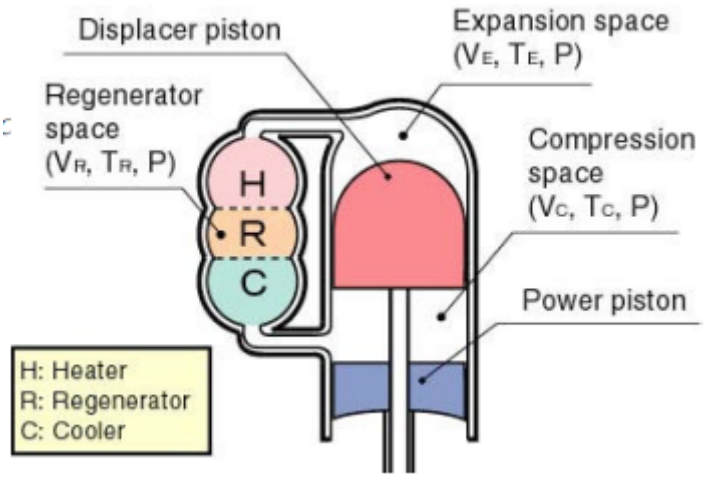
Depending upon the displacer and piston arrangements, there are **3 types of Stirling engines**:

1. **Alpha type Stirling engine:**



Download free eBooks at bookboon.com

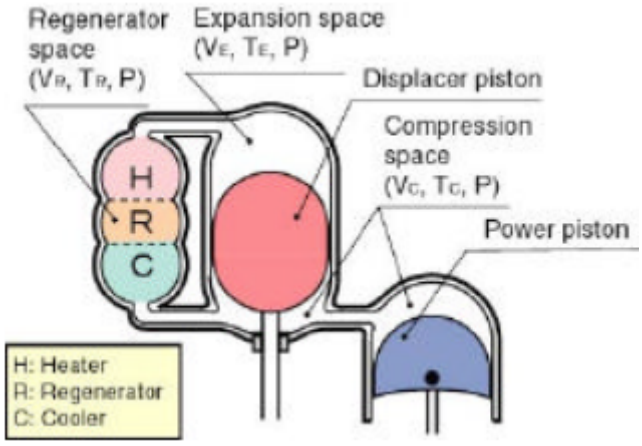
2. **Beta type Stirling engine:**



Download free eBooks at bookboon.com

Click on the ad to read more

3. **Gamma type Stirling engine:**



Assumptions:

a) There is no pressure loss in the heat-exchangers and there are no internal pressure differences.
b) The expansion process and the compression process changes isothermal.
c) Conditions of the working gas is changed as an ideal gas.
d) There is a perfect regeneration.
e) The expansion dead space maintains the expansion gas temperature – T_E, the compression dead space maintains the compression gas temperature – T_C during the cycle.
f) The regenerator gas temperature is an average of the expansion gas temperature – T_E and the compression gas temperature – T_C.
g) The expansion space – V_E and the compression space – V_C changes according a sine curves. **Table below shows symbols used in the Schmidt Theory:**

Download free eBooks at bookboon.com

Name	Symbol	Unit
Engine pressure	P	Pa
Swept volume of expansion piston or displacer piston	V_{SE}	m^3
Swept volume of compression piston or power piston	V_{SC}	m^3
Dead volume of expansion space	V_{DE}	m^3
Regenerator volume	V_R	m^3
Dead volume of compression space	V_{DC}	m^3
Expansion space momental volume	V_E	m^3
Compression space momental volume	V_C	m^3
Total momental volume	V	m^3
Total mass of working gas	m	kg
Gas constant	R	J/kgK
Expansion space gas temperature	T_H	K
Compression space gas temprature	T_C	K
Regenerator space gas temperature	T_R	K
Phase angle	dx	deg
Temperatuer ratio	t	
Swept volume ratio	v	
Dead volume ratio	X	
Engine speed	n	Hz
Indicated expansion energy	W_E	J
Indicated compression energy	W_C	J
Indicated energy	W_i	J
Indicated expansion power	L_E	W
Indicated compression power	L_C	W
Indicated power	L_i	W
Indicated efficiency	e	

Calculation equations for these three types of engines are presented in the above Reference.

For example, calculation equations for an Alpha-type Stirling engine are given below:

The pressure is calculated using an ideal gas method as shown in equation (1).

$$PV = mRT \tag{1}$$

The volumes of the expansion- and compression cylinder at a given crank angle are determined at first.

The momental volumes is described with a crank angle – x. This crank angle is defined as x=0 when the expansion piston is located the most top position (top dead point).

Download free eBooks at bookboon.com

The momental expansion volume – V_E is described in equation (2) with a swept volume of the expansion piston – V_{SE}, an expansion dead volume – V_{DE} under the condition of assumption (g).

$$V_E = \frac{V_{SE}}{2}(1-\cos x)+V_{DE} \tag{2}$$

The momental compression volume – V_C is found in equation (3) with a swept volume of the compression piston – V_{SC}, a compression dead volume – V_{DC} and a phase angle – dx.

$$V_C = \frac{V_{SC}}{2}\{1-\cos(x-dx)\}+V_{DC} \tag{3}$$

The total momental volume is calculated in equation (4).

$$V = V_E + V_R + V_C \tag{4}$$

By the assumptions (a), (b) and (c), the total mass in the engine – m is calculated using the engine pressure – P, each temperature – T , each volume – V and the gas constant – R.

$$m = \frac{PV_E}{RT_E}+\frac{PV_R}{RT_R}+\frac{PV_C}{RT_C} \tag{5}$$

This e-book
is made with
SetaPDF

SETASIGN

PDF components for PHP developers

www.setasign.com

Download free eBooks at bookboon.com

Click on the ad to read more

The temperature ratio – t, a swept volume ratio – v and other dead volume ratios are found using the following equations:

$$t = \frac{T_C}{T_E} \tag{6}$$

$$v = \frac{V_{SC}}{V_{SE}} \tag{7}$$

$$X_{DE} = \frac{V_{DE}}{V_{SE}} \tag{8}$$

$$X_{DC} = \frac{V_{DC}}{V_{SE}} \tag{9}$$

$$X_R = \frac{V_R}{V_{SE}} \tag{10}$$

The regenerator temperature – T_R is calculated in equation (11), by using the assumption (f).

$$T_R = \frac{T_E + T_C}{2} \tag{11}$$

When equation (5) is changed using equation (6)–(10), the total gas mass – m is described in the next equation (12):

$$m = \frac{P}{RT_C}\left(tV_E + \frac{2\tau V_R}{1+t} + V_C\right) \tag{12}$$

Equation (12) is changed in equation (13), using equation (2) and (3).

$$m = \frac{PV_{SE}}{2RT_C}\{S - B\cos(x-a)\} \tag{13}$$

Now,

$$a = \tan^{-1}\frac{v \cdot \sin dx}{t + \cos dx} \tag{14}$$

$$S = t + 2tX_{DE} + \frac{4tX_R}{1+t} + v + 2X_{DC} \tag{15}$$

$$B = \sqrt{t^2 + 2tv\cos dx + v^2} \tag{16}$$

Download free eBooks at bookboon.com

The engine pressure – P is defined as a next equation using equation (13).

$$P = \frac{2mRT_C}{V_{SE}\{S - B\cos(\theta - a)\}} \tag{17}$$

The mean pressure – P_{mean} can be calculated as follows:

$$P_{mean} = \frac{1}{2\pi}\oint P dx = \frac{2mRT_C}{V_{SE}\sqrt{S^2 - B^2}} \tag{18}$$

c is defined in the next equation:

$$c = \frac{B}{S} \tag{19}$$

As a result, the engine pressure – P, based the mean engine pressure – P_{mean} is calculated in equation (20).

$$P = \frac{P_{mean}\sqrt{S^2 - B^2}}{S - B\cos(x - a)} = \frac{P_{mean}\sqrt{1 - c^2}}{1 - c\cdot\cos(x - a)} \tag{20}$$

On the other hand, in the case of equation (17), when cos(x-a) = -1, the engine pressure – P becomes the minimum pressure – P_{min}, the next equation is introduced.

$$P_{\min} = \frac{2mRT_C}{V_{SE}(S + B)} \tag{21}$$

Therefore, the engine pressure – P, based the minimum pressure – P_{min} is described in equation (22).

$$P = \frac{P_{\min}(S + B)}{S - B\cos(x - a)} = \frac{P_{\min}(1 + c)}{1 - c\cdot\cos(x - a)} \tag{22}$$

Similarly, when cos(x-a) = 1, the engine pressure – P becomes the maximum pressure – P_{max}. The following equation is introduced.

$$P = \frac{P_{\max}(S - B)}{S - B\cos(x - a)} = \frac{P_{\max}(1 - c)}{1 - c\cdot\cos(x - a)} \tag{23}$$

The P-V diagram of Alpha-type Stirling engine can be made with above equations.

Download free eBooks at bookboon.com

Indicated Power, Energy and Efficiency:

The indicated energy (area of the P-V diagram) in the expansion and compression space can be calculated analytically with use of the above coefficients. The indicated energy in the expansion space (indicated expansion energy) – W_E(J), based on the mean pressure – P_{mean}, the minimum pressure – P_{min} and the maximum pressure – P_{max} are described in the following equations.

$$W_E = \oint P dV_E = \frac{P_{mean} V_{SE} \pi c \cdot \sin a}{1+\sqrt{1-c^2}} = \frac{P_{\min} V_{SE} \pi c \cdot \sin a}{1+\sqrt{1-c^2}} \cdot \frac{\sqrt{1+c}}{\sqrt{1-c}} = \frac{P_{\max} V_{SE} \pi c \cdot \sin a}{1+\sqrt{1-c^2}} \cdot \frac{\sqrt{1-c}}{\sqrt{1+c}} \quad (52)$$

The indicated energy in the compression space (indicated compression energy) – W_C(J) are described in the next equations.

$$W_C = \oint P dV_C = -\frac{P_{mean} V_{SE} \pi c t \cdot \sin a}{1+\sqrt{1-c^2}} = -\frac{P_{\min} V_{SE} \pi c t \cdot \sin a}{1+\sqrt{1-c^2}} \cdot \frac{\sqrt{1+c}}{\sqrt{1-c}} = -\frac{P_{\max} V_{SE} \pi c t \cdot \sin a}{1+\sqrt{1-c^2}} \cdot \frac{\sqrt{1-c}}{\sqrt{1+c}} \quad (53)$$

The indicated energy per one cycle of this engine – W_i(J) is demanded in the next equations.

$$W_i = W_e + W_c$$

$$= \frac{P_{mean} V_{SE} \pi c (1-t) \sin a}{1+\sqrt{1-c^2}} = \frac{P_{\min} V_{SE} \pi c (1-t) \sin a}{1+\sqrt{1-c^2}} \cdot \frac{\sqrt{1+c}}{\sqrt{1-c}} = \frac{P_{\max} V_{SE} \pi c (1-t) \sin a}{1+\sqrt{1-c^2}} \cdot \frac{\sqrt{1-c}}{\sqrt{1+c}} \quad (54)$$

Deloitte.

Discover the truth at www.deloitte.ca/careers

© Deloitte & Touche LLP and affiliated entities.

Download free eBooks at bookboon.com

Click on the ad to read more

Relations between P_{mean}, P_{min} and P_{max} are determined in the following equations.

$$\frac{P_{min}}{P_{mean}} = \sqrt{\frac{1-c}{1+c}} \tag{55}$$

$$\frac{P_{max}}{P_{mean}} = \sqrt{\frac{1+c}{1-c}} \tag{56}$$

The indicated expansion power – L_E(W), the indicated compression power – L_C(W) and the indicated power of this engine – L_i(W) are defined in the following equations, using the engine speed per one second, n(rps, Hz).

$$L_E = W_E n \tag{57}$$

$$L_C = W_C n \tag{58}$$

$$L_i = W_i n \tag{59}$$

The indicated expansion energy – W_E found equation (52) means an input heat from a heat source to the engine. The indicated compression energy – W_c calculated by equation (53) means a reject heat from the engine to cooling water or air. Then the thermal efficiency of the engine – e is calculated in the next equation.

$$e = \frac{W_i}{W_E} = 1 - t \tag{60}$$

This efficiency equals that of a Carnot cycle which is the highest possible efficiency in every thermal engine.

Note: Example problem illustrated in the above reference is reworked using Mathcad later. See at the end of Prob. 4.2.25.

Download free eBooks at bookboon.com

4.1.10 VUILLEUMIER REFRIGERATOR [1]:

The Vuilleumier refrigerator, first patented by Rudolph Vuilleumier in 1918, is similar to the Stirling refrigerator, except the VM refrigerator uses a "thermal" compressor instead of a mechanical compressor. A schematic of the VM refrigerator and the T-s diagram are shown below:

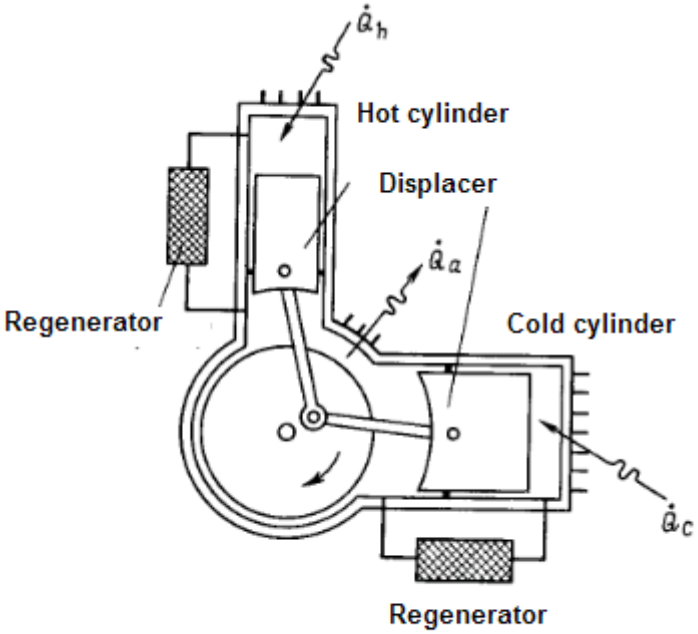


Fig. Schematic of VM refrigerator

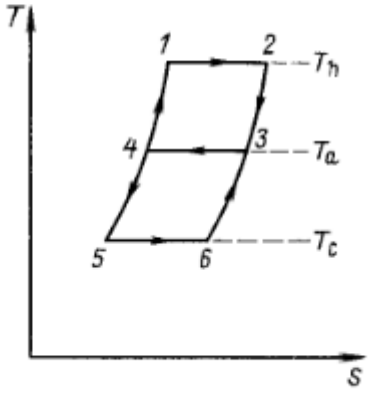


Fig. T-s diagram of VM refrigerator

Working of the refrigerator:

Process 1-2: Heat is added from a high-temperature source to the gas in the "hot" cylinder, and the displacer moves downward to maintain the temperature of the gas constant at T_h.

Process 4-1: At the same time, near-ambient temperature gas flows from the intermediate volume through the regenerator to the hot volume.

Process 2-3: The displacer then moves upward and gas is displaced from the hot volume to the intermediate volume.

Process 3-4: Heat is rejected from the intermediate volume to maintain the temperature of the gas in the volume constant at T_a.

Download free eBooks at bookboon.com

Process 5-6: As the cold displacer is moved to the left, heat is absorbed by the gas in the cold volume from the low-temperature source to maintain the gas temperature constant at T_c.

Process 4-5: At the same time, gas from the intermediate volume flows through the cold regenerator to the cold volume.

Process 6-3: The cold displacer then moves back to the right, and gas is displaced from the cold volume through the cold regenerator to the intermediate volume.

Assuming that all processes are thermodynamically ideal and that the working fluid may be treated as an ideal gas, we get:

$$Q_h = m_h T_h (s_2 - s_1) = m_h R T_h \ln(\upsilon_2 / \upsilon_1)$$

The heat added from the low-temperature source is

$$Q_c = m_c T_c (s_6 - s_5) = m_c R T_c \ln(\upsilon_6 / \upsilon_5)$$

be
your degree
© 2013 Accenture. All rights reserved.
Bring your talent and passion to a global organization at the forefront of business, technology and innovation. Discover how great you can be.
Visit accenture.com/bookboon
Be greater than.
consulting | technology | outsourcing
accenture
High performance. Delivered.
Download free eBooks at bookboon.com
Click on the ad to read more

where m_h and m_c are the mass finally within the hot and cold volumes, respectively, and R is the specific gas constant. Finally, the heat rejected to the intermediate-temperature sink is given by

$$Q_a = (m_h + m_c) T_a (s_4 - s_3) = -(m_h + m_c) R T_a \ln(\upsilon_3 / \upsilon_4)$$

From the temperature-entropy diagram for the cycle, we see that $\upsilon_2 = \upsilon_3 = \upsilon_6$ and $\upsilon_1 = \upsilon_4 = \upsilon_5$.

Because the net heat transfer for the system is zero (there is no external work done on or by the gas, for the ideal system), $Q_h + Q_c + Q_a = 0$, and

$$m_h R T_h \ln(\upsilon_2 / \upsilon_1) + m_c R T_c \ln(\upsilon_2 / \upsilon_1) - (m_h + m_c) R T_a \ln(\upsilon_2 / \upsilon_1) = 0$$

Or, $$m_c / m_h = (T_h - T_a) / (T_a - T_c)$$

Because this system is driven by a heat transfer from a high-temperature source, instead of by mechanical work, the coefficient of performance must be defined a little differently. In this case, the COP is

$$COP = Q_c / Q_h = m_c T_c / m_h T_h$$

Making the substitution for the mass ratio, we find

$$COP = \frac{T_c (T_h - T_a)}{T_h (T_a - T_c)}$$

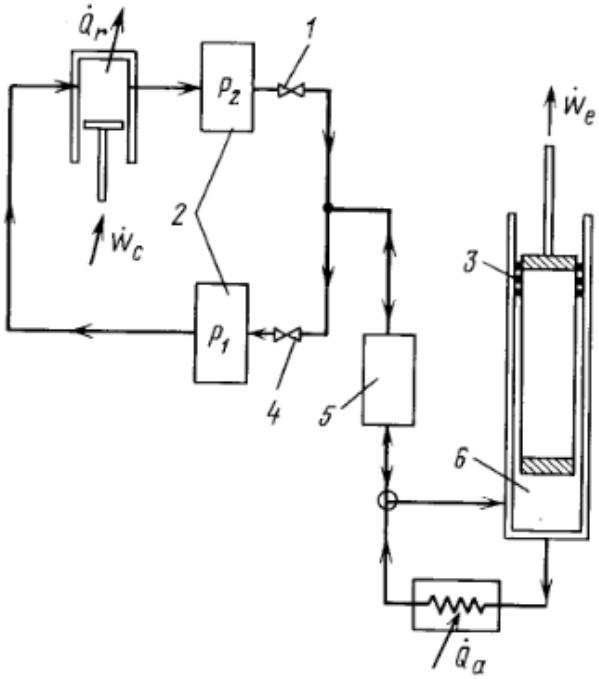
One of the advantages of the Vuilleumier refrigerator is that the thermal input may be provided by solar energy or isotope energy, which makes the VM refrigerator attractive for cryogenic cooling in long-duration space exploration and in applications where mechanical vibration of a drive engine must be avoided.

4.1.11 SOLVAY REFRIGERATOR [1]:

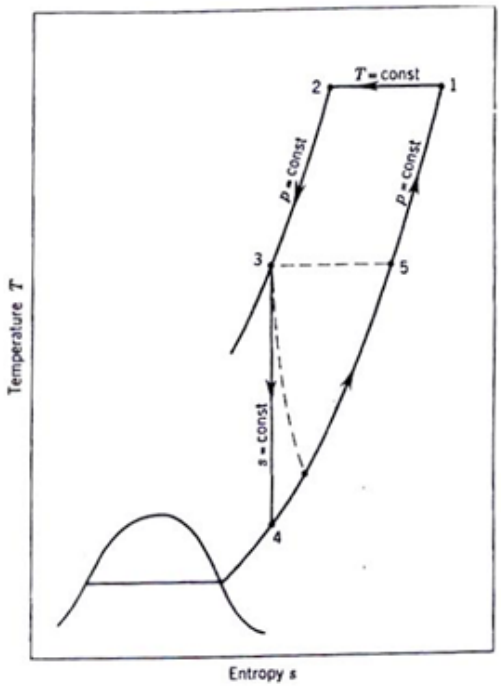
The Solvay refrigerator was invented in Germany in about 1887 (Solvay 1887), and was the first system planned for air liquefaction using an expansion engine.

Download free eBooks at bookboon.com

The Solvay refrigerator is shown schematically, along with the T-s diagram, in Fig. below:



Solvay refrigerator.
1-Inlet valve, 2-Surge volumes, 3-Seal, 4-Exh. valve, 5-Regenerator, 6-Expan. space



T-s diagram for unit mass of gas

The working of the Solvay refrigerator is as follows:

Process 1-2: With the piston at the bottom of its stroke, the inlet valve is opened. The high-pressure gas flows into the regenerator, in which the gas is cooled, and the system pressure is increased from a low pressure P1 to a higher pressure P2'.

Process 2-3: With the inlet valve still open, the piston is raised to draw a volume of gas into the cylinder. The gas has been cooled during its flow through the regenerator.

Process 3-4: The inlet valve is closed, and the gas within the cylinder is expanded (isentropically in the ideal case) to the initial pressure P1 – As the gas expands, it does work on the piston, and energy is removed from the gas as work. The temperature of the gas therefore decreases.

Process 4-5: The exhaust valve is opened, and the piston is lowered to force the cold gas out of the cylinder. During this process, the cold gas passes through a heat exchanger to remove heat from the cooled.

Download free eBooks at bookboon.com

Process 5-1: The gas finally passes out through the regenerator, in which the cold gas is warmed back to room temperature.

Assuming that the work output during the expansion process is utilized in the compression process, the net work requirement for this system is given by

$$-W_{net}/m = \left[T_2(s_1 - s_2) - (h_1 - h_2)\right] / \eta_{c,0} - \eta_{e,m}\eta_{ad}(h_3 - h_4)$$

where the first term represents the compressor work and the second term represents the work output during the expansion process. The enthalpy h_4 is the value that would be achieved at the end of an isentropic expansion from point 3 to the pressure P1, at point 4. The energy removed from the low-temperature Source is given by

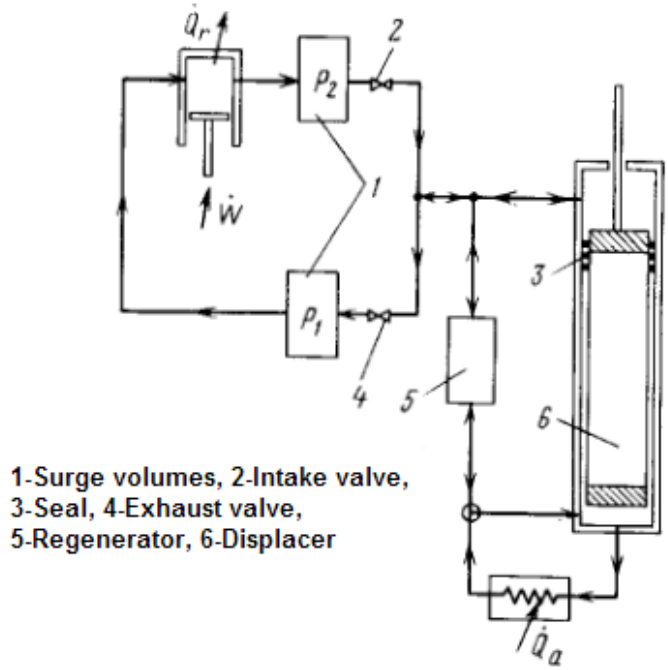
$$Q_a/m = h_5 - h_4' = h_5 - h_4 - (1 - \eta_{ad})(h_3 - h_4)$$

The piston is constructed of a poor heat conductor, such as micarta, so that it may be sealed at the warm end, which helps to avoid the problem of a low-temperature moving seal.

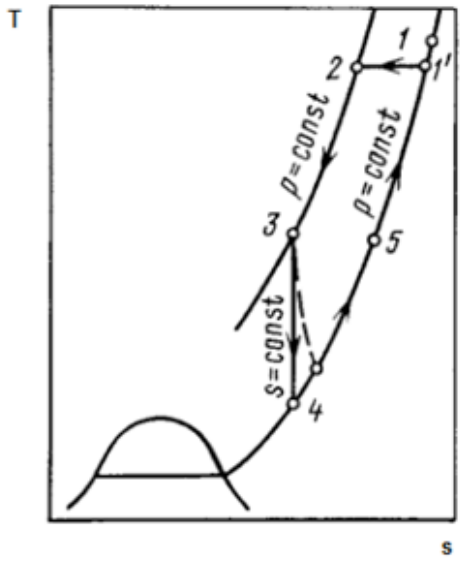
The Wake
the only emission we want to leave behind
Low-speed Engines Medium-speed Engines Turbochargers Propellers Propulsion Packages PrimeServ
The design of eco-friendly marine power and propulsion solutions is crucial for MAN Diesel & Turbo.
Power competencies are offered with the world's largest engine programme – having outputs spanning
from 450 to 87,220 kW per engine. Get up front!
Find out more at www.mandieselturbo.com
Engineering the Future – since 1758.
MAN Diesel & Turbo
MAN
Download free eBooks at bookboon.com
Click on the ad to read more

4.1.12 GIFFORD-MCMAHON (GM) REFRIGERATOR [1, 12, 13, 14]:

A schematic diagram of the Gifford-McMahon refrigerator and T-s diagram for a unit mass of the working are shown below. This system consists of a compressor, a cylinder closed at both ends, a displacer within the cylinder, and a regenerator. This system differs from the Solvay refrigerator in that no work is transferred from the system during the expansion process. The displacer serves the purpose of moving the gas from one expansion space to another and would do zero net work in the ideal case of zero pressure drop in the regenerator.



G-M cycle refrigerator.



T-s diagram for unit mass of gas

Working of the Gifford-McMahon refrigerator is as follows:

Process 1-2: With the displacer at the bottom of the cylinder, the inlet valve is opened and the pressure within the upper expansion space is increased from a low pressure p1 to a higher pressure p2.

Process 2-3: With the inlet valve still open and the exhaust valve closed, the displacer is moved to the top of the cylinder. The gas that was originally in the upper expansion space moves down through the regenerator to the lower expansion space. Because the gas is cooled as it passes through the regenerator, it will decrease in volume, and some more gas will be drawn in through the inlet valve during this process to maintain a constant pressure within the system.

Download free eBooks at bookboon.com

Process 3-4: With the displacer at the top of the cylinder, the inlet valve is closed and the exhaust valve is opened, thus allowing the gas within the lower expansion space to expand to the initial pressure p1. The gas that is finally within the lower expansion space does work to push out the gas that leaves during this process; therefore, energy is removed as work from the gas finally left in the lower expansion space. This causes the gas in the lower expansion space to drop to a low temperature.

Process 4-5: The low-temperature gas is forced out of the lower expansion space by moving the displacer downward to the bottom of the cylinder. This cold gas flows through a heat exchanger in which heat is transferred to the gas from the low-temperature source.

Process 5-1: The gas flows from the heat exchanger through the regenerator, and the gas is warmed back to near ambient temperature.

The net work requirement for this system is given by

$$-W/m = \left[T_1(s_1 - s_2) - (h_1 - h_2)\right] / n_{c,\,0}$$

The energy removed from the low-temperature source is given by

$$Q_a / m = (m_e / m)(h_5 - h'_4) = \eta_{ad}\,(m_c / m)(h_5 - h_4)$$

where m_e, is the mass of gas within the lower expansion space at the end of the expansion process 3-4, and m is the total mass of gas compressed. Because the volume of the expansion space remains constant during the expansion process. the mass ratio m_e/m may be written in terms of the density ratio:

$$m_e / m = \rho'_4 / \rho_3$$

There are several factors that contribute to a loss in performance of the Gifford-McMahon refrigerator: regenerator ineffectiveness, thermal conduction down the displacer and its housing, "shuttle" heat transfer, and the finite volume within the regenerator.

Measurements of the performance of a small infrared cooler operatll1g on the Gifford-McMahon cycle showed that the actual refrigeration effect was approximately 59 percent of the ideal refrigeration effect.

Download free eBooks at bookboon.com

4.1.13 REGENERATORS USED IN SOLVAY AND GIFFORD-MCMAHON REFRIGERATORS [1]:

The regenerator is a critical component, as in the case of the Philips refrigerator. For an efficient refrigerator, the regenerator effectiveness should be 98 percent or better. Punched copper or brass screens were used as the regenerator packing material. To reduce the heat conduction along the length of the regenerator, the punched screens were separated by a coil of stainless-steel wire. See fig. below:

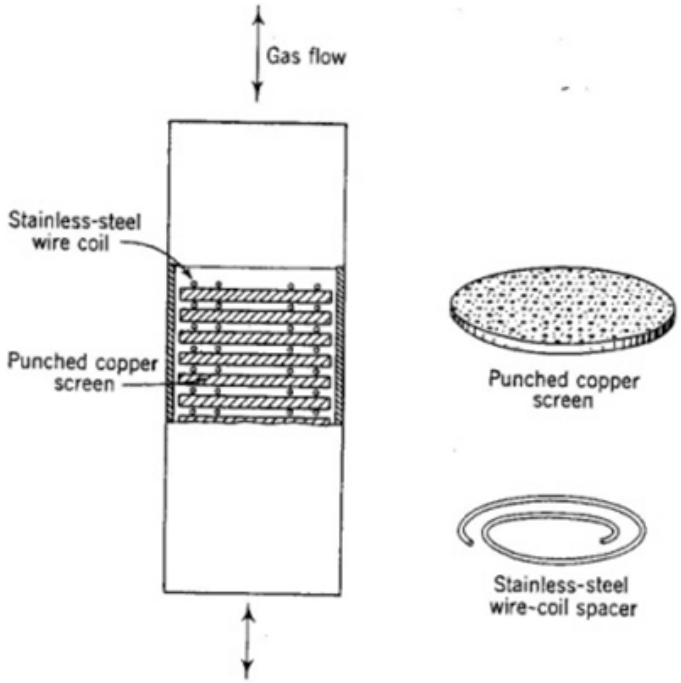


Fig. Punched wire screen regenerator.

For very low temperature regenerators, lead may be used instead of copper because lead has a 'higher specific heat at low temperature due to its lower Debye temperature.

Design equations for Regenerators [1]:

Ref.[1] distinguishes between two types of regenerators:

i) **those used for Stirling, Solvay and G-M refrigerators**, where inlet temps of hot and cold fluids are not constant during the heating and cooling periods, and the particle 'dwell time' in the regenerator is much more (about twice) than the heating or cooling period.
ii) **those used in air liquefaction plants and gas turbine systems**, where the particle dwell time is less than about one-tenth of the heating or cooling period.

Download free eBooks at bookboon.com

We shall briefly present the design equations as given in Ref.[1] for both these types of regenerators:

i) **For regenerators used in Stirling, Solvay and G-M refrigerators:**

a) **For random stacked, spherical particle matrix:**

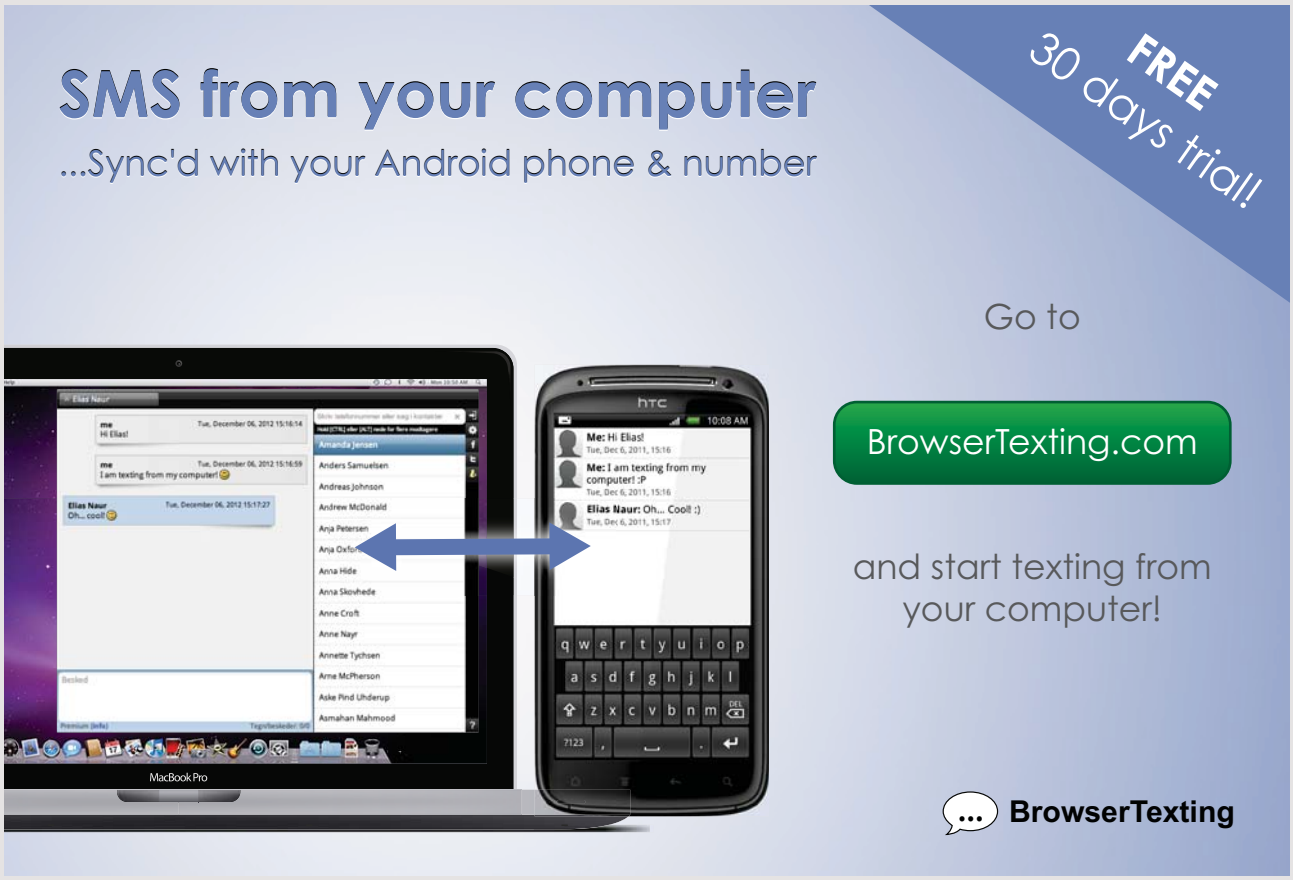
For heat transfer and pressure drop calculations:

$$j_H = \frac{h_c \cdot Pr^{\frac{2}{3}}}{G \cdot cp} = 0.255 \cdot (e_v \cdot Re)^{\frac{-1}{3}}$$

$$f = \frac{1 - e_v}{e_v} \cdot \left[\frac{570 \cdot (1 - e_v)^2}{e_v \cdot Re} + 3.5 \right]$$

$$G = \frac{mdot}{A_{ff}} \qquad A_{ff} = \frac{e_v \cdot V_0}{L}$$

$$A = \frac{6 \cdot (1 - e_v) \cdot V_0}{D_s}$$



Download free eBooks at bookboon.com
Click on the ad to read more

Reynolds No. is based on sphere dia D_s.

Mass velocity, G = m_dot / A_{ff} where A_{ff} = free flow area = e_v. V_0 /L

And, e_v is the **porosity** of matrix, V_0 is the total volume and L = matrix length.

Heat transfer surface area for spherical matrix, A = 6. (1-e_v). V_0 /D_s

Pressure drop is determined from:

$\Delta P = (f.L / D_s).(G^2 / 2.\rho)$ … Pa

b) **For stacked screen matrix:**

$$n = 0.37 + \frac{0.0136 \cdot e_v}{1 - e_v} \qquad jH = \frac{0.2 \cdot Re^{-n}}{1 - e_v} \quad \text{...Colburn j-factor}$$

$$f = \frac{860 \cdot (1 - e_v)}{Re} + \frac{2.2 \cdot e_v}{Re^{0.1}} \quad \text{...friction factor}$$

Reynolds No. is based on equivalent dia, given by:

$$D_e = \frac{4 \cdot A_{ff} \cdot L}{A} = \frac{4 \cdot e_v \cdot V_0}{A}$$

For heat transfer with Stacked screens, another relation due to Ackermann [*Ref: R&D Technical Report ECOM-3245, (1970)*] *is*:

$j_H = 0.68. (N_{Re})^{-0.407}$

Remember:

Reynolds No, $N_{Re} = (G.D_e) / \mu$

Colburn j-factor, j_H = St. $Pr^{(2/3)}$

Stanton No., St = h / (G.cp)

Prandtl No., Pr = (cp. μ)/ k

Download free eBooks at bookboon.com

ii) **For regenerators used in Air liquefaction plants and Gas turbine systems:**

Ref.[1] presents **two approaches**: one, the NTU approach, similar to the one used in Heat exchanger design; and, second, Hausen's method, called Lambda – PI (or, $\Lambda - \prod$) method.

a) **NTU approach:**

We define:

$$NTU_0 = \frac{1}{C_{min}} \cdot \left(\frac{1}{h_h \cdot A_h} + \frac{1}{h_c \cdot A_c} \right)^{-1}$$

$$C_R := \frac{C_{min}}{C_{max}}$$

where h_h and A_h are the heat transfer coeff. and area for the heating part of the cycle, and h_c and A_c are the respective quantities for the cooling part of the cycle.

For valved regenerators used in Air liquefaction plants and those used in refrigerators, A_h and A_c are equal. For rotary regenerators used in Gas turbine systems, A_h and A_c may be different.

Download free eBooks at bookboon.com
Click on the ad to read more

For regenerators, another quantity, called **'matrix capacity rate ratio'** is defined:

$$C_m = \frac{ms \cdot cs}{P0 \cdot C_{min}}$$

P_0 = total period of regenerator, i.e.

$P_0 = P_h + P_c$

And, **regenerator effectiveness** is defined as:

ε = Q_dot / (C_{min}. ($T_{h1} - T_{c1}$))

In the above, T_{h1} and T_{c1} are the inlet temps of hot and cold fluids respectively, and Q_dot is the average heat transfer rate.

Values of effectiveness for a counter-flow regenerator as a function of NTU_0 and C_m when $C_R = (C_{min} / C_{max}) = 1$ are presented in the following Table:

Effectiveness of a counterflow regenerator as function of NTU_0 and C_m for C_R = Cmin/Cmax = 1

	Matrix capacity rate ratio, C_m							
N_{tu0}	0,8	1,0	1,25	1,5	2,0	3,0	5,0	∞
0	0	0	0	0	0	0	0	0
0,5	0,315	0,322	0,326	0,328	0,330	0,332	0,333	0,3333
1,0	0,449	0,467	0,478	0,485	0,491	0,496	0,499	0,5000
1,5	0,521	0,548	0,566	0,576	0,586	0,594	0,598	0,6000
2,0	0,566	0,601	0,623	0,636	0,649	0,659	0,664	0,6667
2,5	0,599	0,639	0,664	0,679	0,694	0,705	0,711	0,7143
3,0	0,622	0,667	0,696	0,712	0,728	0,740	0,746	0,7500
3,5	0,642	0,690	0,721	0,738	0,755	0,767	0,774	0,7778
4,0	0,659	0,709	0,741	0,759	0,776	0,789	0,796	0,8000
4,5	0,673	0,724	0,758	0,776	0,794	0,807	0,814	0,8182
5,0	0,685	0,738	0,772	0,791	0,809	0,822	0,829	0,8333
5,5	0,696	0,749	0,785	0,803	0,821	0,834	0,842	0,8462
6,0	0,705	0,759	0,796	0,814	0,832	0,845	0,853	0,8571
6,5	0,713	0,768	0,805	0,824	0,842	0,855	0,862	0,8667
7,0	0,721	0,776	0,814	0,833	0,850	0,863	0,870	0,8750
7,5	0,728	0,784	0,822	0,840	0,858	0,871	0,878	0,8824
8,0	0,734	0,790	0,829	0,847	0,865	0,877	0,884	0,8889
8,5	0,740	0,796	0,835	0,854	0,871	0,883	0,890	0,8947
9,0	0,746	0,802	0,841	0,859	0,876	0,888	0,895	0,9000
9,5	0,751	0,807	0,846	0,864	0,881	0,893	0,900	0,9048
10,0	0,756	0,811	0,851	0,869	0,886	0,898	0,904	0,9091
20,0	0,796	0,865	0,906	0,922	0,935	0,943	0,948	0,9524
30,0	0,816	0,889	0,928	0,943	0,947	0,961	0,964	0,9677
40,0	0,826	0,903	0,942	0,955	0,963	0,971	0,974	0,9756
50,0	0,834	0,914	0,951	0,962	0,970	0,975	0,978	0,9804
60,0	0,844	0,920	0,957	0,968	0,974	0,980	0,982	0,9836
90,0	0,862	0,935	0,969	0,977	0,982	0,986	0,987	0,9890
100,0	0,867	0,939	0,971	0,979	0,984	0,987	0,989	0,9901
500,0	0,891	0,974	0,993	0,995	0,996	0,997	0,998	0,9980

Download free eBooks at bookboon.com

For a regenerator under 'unbalanced operation' (i.e. when C_R is *not equal* to 1), effectiveness is calculated as follows:

First, 'effective values' of NTU and matrix capacity rate ratios are calculated from the following expressions:

$$NTU_e = \frac{2 \cdot NTU_0 \cdot C_R}{(1 + C_R)}$$

$$C_{me} = \frac{2 \cdot C_m \cdot C_R}{(1 + C_R)}$$

Using these 'effective values', effectiveness of a balanced regenerator, ε_1 is calculated from the above Table.

Then, following parameter is determined:

$$X = \frac{1 + C_R}{2 \cdot C_R} \cdot \frac{\varepsilon_1}{1 - \varepsilon_1} \cdot (1 - C_R)$$

Finally, regenerator effectiveness is calculated from:

$$\varepsilon = \frac{1 - \exp(-X)}{1 - C_R \cdot \exp(-X)}$$

b) **Hausen's method, called Lambda – PI ($\Lambda - \prod$) approach:**

'Reduced length' and 'Reduced period' are defined:

Reduced length: $\Lambda_h = \frac{h_h \cdot A_h}{mdot_h \cdot c_h}$ and, $\Lambda_c = \frac{h_c \cdot A_c}{mdot_c \cdot c_c}$

Reduced period: $\Pi_h = \frac{h_h \cdot A_h \cdot P_h}{m_{sh} \cdot c_s}$ and, $\Pi_c = \frac{h_c \cdot A_c \cdot P_c}{m_{sc} \cdot c_s}$

Regenerator is said to be *symmetric*, if:

$$\prod_h = \prod_c$$

And, regenerator is *balanced*, if:

$$\Lambda_h / \prod_h = \Lambda_c / \prod_c$$

Download free eBooks at bookboon.com

Values of effectiveness for a symmetric and balanced regenerator are presented in the following Table:

Λ	Π				
	0	5	10	15	20
0	0	0	0	0	0
5	0,7143	0,639	0,469	0,326	0,250
10	0,8333	0,809	0,738	0,610	0,494
15	0,8824	0,866	0,840	0,784	0,693
20	0,9091	0,900	0,886	0,856	0,811
25	0,9259	0,920	0,911	0,895	0,871
30	0,9375	0,933	0,927	0,918	0,903
35	0,9459	0,943	0,939	0,932	0,922
40	0,9524	0,950	0,947	0,942	0,935
45	0,9574	0,955	0,953	0,949	0,944
50	0,9615	0,961	0,958	0,955	0,951
100	0,9804	0,980	0,979	0,979	0,878
200	0,9901	0,990	0,989	0,989	0,989

Brain power
By 2020, wind could provide one-tenth of our planet's electricity needs. Already today, SKF's innovative know-how is crucial to running a large proportion of the world's wind turbines.
Up to 25 % of the generating costs relate to maintenance. These can be reduced dramatically thanks to our systems for on-line condition monitoring and automatic lubrication. We help make it more economical to create cleaner, cheaper energy out of thin air.
By sharing our experience, expertise, and creativity, industries can boost performance beyond expectations.
Therefore we need the best employees who can meet this challenge!
The Power of Knowledge Engineering
Plug into The Power of Knowledge Engineering.
Visit us at www.skf.com/knowledge
SKF
Download free eBooks at bookboon.com
Click on the ad to read more

And, Table is contd. below:

Λ	Π					
	25	30	35	40	45	50
0	0	0	0	0	0	0
5	0,202	0,167	0,143	0,125	0,112	0,100
10	0,402	0,333	0,284	0,250	0,222	0,200
15	0,585	0,498	0,430	0,375	0,332	0,300
20	0,756	0,651	0,578	0,500	0,443	0,400
25	0,827	0,770	0,698	0,620	0,555	0,500
30	0,881	0,845	0,797	0,727	0,654	0,598
35	0,907	0,888	0,856	0,810	0,754	0,692
40	0,925	0,912	0,891	0,865	0,820	0,773
45	0,938	0,928	0,914	0,898	0,871	0,835
50	0,946	0,939	0,929	0,915	0,897	0,875
100	0,977	0,976	0,975	0,973	0,972	0,970
200	0,989	0,989	0,988	0,988	0,988	0,988

For an unbalanced and unsymmetric regenerator, to calculate effectiveness, following method is suggested by Hausen:

First, the harmonic mean values of reduced length and reduced period are calculated from:

$$\frac{2}{\Pi_m} = \frac{1}{\Pi_h} + \frac{1}{\Pi_c}$$

$$\frac{2}{\Lambda_m} = \frac{1}{\Pi_m}\cdot\left(\frac{\Pi_h}{\Lambda_h} + \frac{\Pi_c}{\Lambda_c}\right)$$

Using these mean values, effectiveness ε_1 is calculated from the above Table.

Then, the effectiveness is calculated from the following (as we did in the case of NTU-effectiveness approach, earlier):

$$X = \frac{1 + C_R}{2\cdot C_R}\cdot\frac{\varepsilon_1}{1 - \varepsilon_1}\cdot(1 - C_R)$$

$$\varepsilon = \frac{1 - \exp(-X)}{1 - C_R\cdot\exp(-X)}$$

Note: From the above Tables, we see that *for a large effectiveness, the regenerator should have* a large number of transfer units and a large matrix capacity rate ratio, or a large reduced length and a small reduced period, i.e. there should be a large heat transfer coeff., large surface area, a large product of regenerator mass and specific heat (m_s. cp), and a small operating period, P_0 or a large frequency of switching, $f = 1/P_0$.

Download free eBooks at bookboon.com

4.1.14 AN EXAMPLE OF COMMERCIAL GM CYCLE CRYOCOOLERS: (FROM BROCHURE OF M/S OXFORD CRYOSYSTEMS, OXFORD, U.K.):

(Ref: http://www.oxcryo.com/wp/wp-content/uploads/2014/03/Coolstar-Brochure.pdf)

There are two types: Single stage (up to 77 K) and two stage cryocoolers (up to 20 K):

Single Stage Coldheads: 0/12 and 0/40:

The Coolstar single stage coldheads comprise the 0/12 and 0/40 models, respectively delivering cooling powers of 12 and 40 W at 77 Kelvin. These compact models have the major advantage of being some of the smallest and most cost-effective GM heads available, and can thus be installed in systems where space is at a premium. The coldheads function independently of orientation, and can be used with any Coolstar Cryodrive Compressor.

Two Stage Coldheads: 2/9 and 6/30:

The Coolstar two stage coldheads comprise the 2/9 model (delivering cooling power of 2 W at 20 K and 9 W at 77 K simultaneously) and 6/30 model (delivering cooling power of 6 W at 20 K and 30 W at 77 K simultaneously). Both of these coldheads can reach a base temperature of 10 K. These efcient coolers are still compact, lending themselves to a wide range of applications. The coldheads function independently of orientation, and should be used with the 2.0 kW or 3.0 kW Coolstar Cryodrive compressor, depending on application.

Photos:

Download free eBooks at bookboon.com

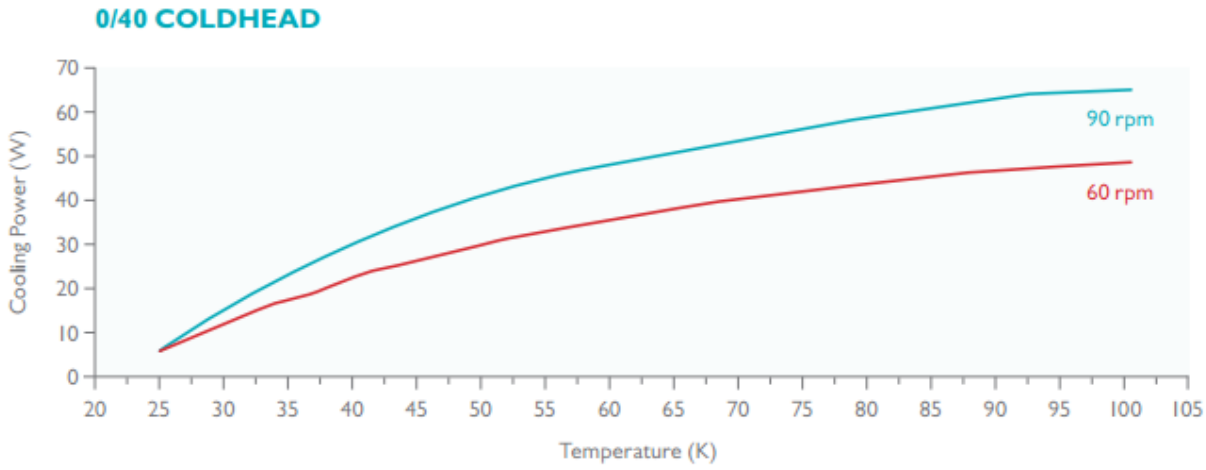
Specifications:

	Coolstar 2/9	Coolstar 6/30	Coolstar 0/12	Coolstar 0/40
Base Temperature (Unloaded)	10 K	10 K	30 K	30 K
Nominal Cooling Power at 20 K			-	-
Normal Speed (72 rpm)	2 W	6 W	-	-
High Speed (90 rpm)	2.4 W	7 W	-	-
Nominal Cooling Power at 77 K				
Normal Speed (72 rpm)	9 W	28 W	12 W	42 W
High Speed (90 rpm)	11 W	30 W	17 W	57 W
Time to cooldown bare coldhead				
to 20 K	<16 min	<18 min		
to 77 K			<11 min	<11 min
Increase in cooldown time for each 100 g Copper added	14 min	5.2 min	6 min	2 min
Weight	2.5 kg	6.1 kg	2.4 kg	5.6 kg
Dimensions	278 x 129 x 150mm	344 x 155 x 207mm	185 x 123 x 150mm	219 x 155 x 207mm

Download free eBooks at bookboon.com
Click on the ad to read more

Electrical Power (±6%)	Only single-phase power required 200 V, 220 V, 240 V 50 Hz or 200 V, 208 V, 220 V, 60 Hz
Typical Running current:	
Cryodrive 2.0	11 A
Cryodrive 3.0	13 A
Typical Helium Pressure	
Running	22.5 Bar
Static	16.5 Bar
Operating Temperature	4-38°C
Typical Cooling Water Flow	5 l/min at 18°C
Weight + Dimensions	80 kg, 451 x 556 x 448mm
Standard Helium Hose and Coldhead Cable Length	3 m
Cooling Water Connection	½" ID nozzle

Typical cooling powers are presented as follows:



Refrigerators for temperatures below 2 K:

4.1.15 MAGNETIC COOLING [1]:

By reducing pressure over boiling Helium-4 we can attain about 1 K (at about 0.12 torr) and by pumping over boiling Helium-3, the limit is about 0.6 K (at about 0.545 torr).

Giaque and Debye suggested that we can attain lower temperatures by using a paramagnetic substance instead of a gas or liquid, and a magnetic field instead of expansion. In the absence of a magnetic field, the dipoles of a paramagnetic substance are randomly oriented even at low temperature, and when a magnetic field is applied at constant temp, they tend to align themselves, reducing the entropy. This is similar to compressing a gas isothermally and reducing the entropy. Now, if the magnetic field is removed reversibly and adiabatically (analogous to isentropic expansion of a gas), to maintain the entropy constant, the temp must decrease. This process is called **'adiabatic demagnetization'**.

Download free eBooks at bookboon.com

A schematic of an apparatus to carry out the adiabatic demagnetization process is shown in Fig. below. A paramagnetic salt pellet is suspended in a chamber by silk or nylon threads. This chamber is initially filled with gaseous helium, and the chamber is then immersed in a liquid-helium (LHe) bath. The LHe is boiling under reduced pressure, so its temperature and the temperature of the paramagnetic salt are about 1 K. The helium bath is surrounded by a liquid-hydrogen or liquid-nitrogen shield to reduce the heat transfer from ambient to the helium bath. This entire assembly is placed between the poles of a powerful electromagnet, which is shaped so that the field of the magnet is concentrated around the salt pellet. The magnetic field is turned on and maintained for about an hour to allow the heat of magnetization (similar to the heat of compression for a gas) to be conducted to the helium bath by the gaseous helium in the small chamber, thereby maintaining the salt at its original temperature. When thermal equilibrium is attained, the gaseous helium (which is called an exchange gas) is pumped away to thermally isolate the paramagnetic salt. The magnetic field is then removed, and the temperature of the salt drops to a very low value. Temperatures as low as 0.0014 K or 1.4 mK have been attained by this method, according to de Klerk, Steen land, and Gorter (1950).

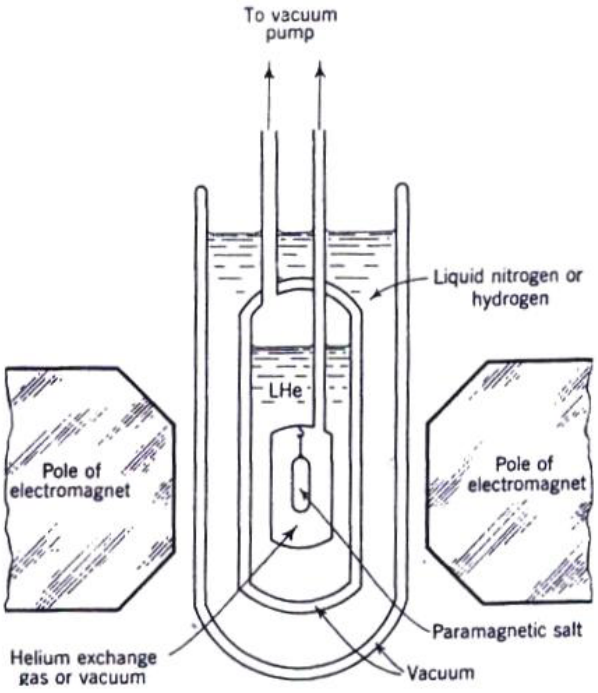


Fig. Apparatus for carrying out an adiabatic demagnetization process.

Download free eBooks at bookboon.com

This process of adiabatic demagnetization will work only for very low temperatures because of the magnitude of the lattice thermal effects at temperatures much above 2 K or 3 K. The lattice entropy must be much smaller than the entropy associated with the magnetic dipoles of the paramagnetic material if a significant temperature change is to be achieved. At very low temperatures, the lattice entropy is given by

$$s \text{ (lattice)} = 77.9\ R(T/\theta_D)^3$$

As we shall see later, the maximum dipole entropy for a simple spin system (S = ½) is given by

$$s(\text{dipole}, H = 0) = R \ln 2$$

To ensure the success of the adiabatic demagnetization process, we should want the lattice, entropy to be 1 percent or so of the dipole entropy. The temperature for this condition to be true can he found from eqn.

$$77.9\ R(T/\theta_D)^3 = 0.01.\ R \ln 2$$

LIGS University
based in Hawaii, USA
is currently enrolling in the Interactive Online BBA, MBA, MSc, DBA and PhD programs:
- enroll by October 31st, 2014 and
- save up to 11% on the tuition!
- pay in 10 installments / 2 years
- Interactive Online education
- visit www.ligsuniversity.com to find out more!

Note: LIGS University is not accredited by any nationally recognized accrediting agency listed by the US Secretary of Education.
More info here.

Download free eBooks at bookboon.com
Click on the ad to read more

or, solving for the upper limiting temperature T_0,

$$T_0 = \theta_D \cdot \left(\frac{\ln(2)}{7790}\right)^{\frac{1}{3}} = 0.0446 \cdot \theta_D$$

This result indicates that a materia1 with a high Debye temperature θ_D would be advantageous for magnetic cooling.

4.1.16 THERMODYNAMICS OF MAGNETIC COOLING [1]:

The magnetic process may be analyzed thermodynamically if, in order to simplify the situation, we consider pressure and volume changes small enough to be neglected. In this case, we may write

$$T\,ds = du - \mu_0 H\,dI$$

where $\mu_0 = 4\pi \times 10^{-7}$ T–m/A = permeability of free space in SI units, H is the magnetic field intensity, A/m, and I is the magnetic moment per unit mass, A-m^2/kg The quantity $-\mu_0 H\,dI$ represents the magnetic work per unit mass corresponding to the volume-change work + p dv for a pure substance.

For pure substance, it can be shown from thermodynamic reasoning (Van Wylen and Sonntag, 1976) that:

$$Tds = dh - \upsilon dp = c_p dT - T\left(\frac{\partial \upsilon}{\partial T}\right)_p dp$$

The analogous expression for a paramagentic substance can be obtained by replacing the specific volume □ by the magnetic moment per unit mass I and by replacing the pressure p by the quantity $\mu_0 H$:

$$Tds = c_H dT + \mu_0 T\left(\frac{\partial I}{\partial T}\right)_H$$

where c_H is the specific heat at constant magnetic field intensity (analogous to c_p for a pure substance).

Download free eBooks at bookboon.com

For the adiabatic demagnetization process, the entropy of the paramagnetic material remains constant; therefore, ds = 0. Making this substitution into above eqn., we may solve for the differential temperature change due to a differential change in the magnetic field intensity while entropy remains constant.

$$\mu_M = \left(\frac{\partial T}{\partial H}\right) = -\frac{\mu_0 T}{c_H}\left(\frac{\partial I}{\partial T}\right)_H$$

$\mu_M = (\partial T/\partial H)_s$ is known as 'magnetocaloric coeff.' and is analogous to the isentropic expansion coeff. for a pure substance.

When the paramagnetic ions do not interact much and when the temp is not too low, or the magnetic field is not too large, the paramagnetic material obeys the **Curie law**, viz.

I = C. H / T where C is the Curie constant, which depends on the material.

See Table below:

Table 5.4. Curie constants for paramagnetic salts

Paramagnetic salt	Ionic Weight, *M* g/mol or lb_m/mol	Density kg/m³	Density lb_m/ft^3	Gas Constant, *R* J/kg-K	Gas Constant, *R* Btu/lb_m-°R	Curie Constant, *C* K-m³/kg
Cerium ethyl sulfate	678			12.263	2.93×10^{-3}	25.1×10^{-6}
Cerium magnesium nitrate	765			10.868	2.60×10^{-3}	5.22×10^{-6}
Chromium methylammonium alum	492	1645	103	16.899	4.04×10^{-3}	30.5×10^{-6}
Chromium potassium alum	499	1830	114	16.662	3.98×10^{-3}	45.8×10^{-6}
Copper potassium sulfate	442	2220	139	18.811	4.49×10^{-3}	12.21×10^{-6}
Gadolinium sulfate	373	3010	188	22.290	5.32×10^{-3}	263.3×10^{-6}
Iron ammonium alum	482	1710	107	17.250	4.12×10^{-3}	114.1×10^{-6}
Manganese ammonium sulfate	391	1830	114	21.264	5.08×10^{-3}	149.2×10^{-6}
Titanium cesium alum	589	2000	125	14.116	3.37×10^{-3}	4.21×10^{-6}

By permission from M. W. Zemansky, *Heat and Thermodynamics*. 1957.

4.1.17 MAGNETIC MOMENT AND ENTROPY OF PARAMAGNETIC MATERIALS [1]:

Brillouin developed following expression for magnetic moment of paramagnetic material:

I = ½ ng$\square_B$ B($\square$)

where B($\square$) is called **Brillouin function** and is given by:

$$B(\xi) = (2 \cdot J + 1) \cdot \coth((2 \cdot J + 1) \cdot \xi) - \coth(\xi)$$

where

$$\xi = \frac{g \cdot \mu_B \cdot \mu_0 \cdot H}{2 \cdot k \cdot T}$$

Download free eBooks at bookboon.com

Each term in the above equations is explained below:

J … is an atomic constant which gives a measure of the angular momentum of the ions = $(h/2.\pi).[J.(J + 1)]^{1/2}$ where h is the Planck's constant.

g … is another atomic constant, called 'Lande splitting factor' or 'spectroscopic splitting factor'.

TURN TO THE EXPERTS FOR
SUBSCRIPTION CONSULTANCY

Subscrybe is one of the leading companies in Europe when it comes to innovation and business development within subscription businesses.

We innovate new subscription business models or improve existing ones. We do business reviews of existing subscription businesses and we develope acquisition and retention strategies.

Learn more at linkedin.com/company/subscrybe or contact
Managing Director Morten Suhr Hansen at mha@subscrybe.dk

SUBSCRYBE - to the future

Download free eBooks at bookboon.com
Click on the ad to read more

Some values of atomic constants are given in following Table:

Paramagnetic salt	g	J
Cerium ethyl sulfate	1.7	2/2
Cerium magnesium nitrate	1.84	1/2
Chromium methylammonium alum	2.00	3/2
Chromium potassium alum	1.97	3/2
Chromium potassium sulfate	2.14	1/2
Gadolinium sulfate	1.992	7/2
Iron ammonium alum	2.00	5/2
Magnesium ammonium alum	2.06	5/2
Titanium cesium alum	1.89	1/2

μ_B …. Bohr magneton … a natural constant, and

$\mu_B := 0.9273 \cdot 10^{-23}$ ...A-m^2 ... Bohr magneton

μ_0 …. Permeability of free space… and

$\mu_0 := 4 \cdot \pi \cdot 10^{-7}$ ...J/A-m^2 ... permeability of free space

k ….. is the Boltzmann constant, and

$k := 1.3805 \cdot 10^{-23}$ ...J/K ... Boltzmann const.

n …. is the number of ions per unit mass, and

$$n := \frac{N_0}{M},$$

where

$N_0 := 6.023 \cdot 10^{23}$ mol^-1 ... Avogadro's No. And, M = mol. wt. or ionic weight

Download free eBooks at bookboon.com

Expression for entropy of paramagnetic salt (neglecting lattice entropy contribution):

$$s = R \cdot \ln\left[\frac{\sinh((2 \cdot J + 1) \cdot \xi)}{\sinh(\xi)}\right] - R \cdot \xi \cdot B(\xi)$$

In the case of zero magnetic field: $s = R \cdot \ln(2 \cdot J + 1)$ ...for H = 0

Also: $s = R \cdot (2 \cdot J \cdot \xi) - R \cdot \xi \cdot 2 \cdot J = 0$ .. for H/T very large.

Brillouin function and entropy of paramagnetic materials are given the following Table:

Remember: $\xi = \frac{g \cdot \mu_B \cdot \mu_0 \cdot H}{2 \cdot k \cdot T}$

Brillouin function and entropy for paramagnetic materials

ξ	$J = 1/2$		$J = 3/2$		$J = 5/2$		$J = 7/2$	
	$B(\xi)$	s/R	$B(\xi)$	s/R	$B(\xi)$	s/R	$B(\xi)$	s/R
0,0	0,00000	0,69315	0,00000	1,38629	0,00000	1,79176	0,0000	2,07944
0,1	0,09967	0,68817	0,49442	1,36171	1,1388	1,73550	2,0142	1,98082
0,1	0,09967	0,68817	0,49442	1,36171	1,1388	1,73550	2,0142	1,98082
0,2	0,19738	0,67354	0,95727	1,29266	2,1307	1,58838	3,6134	1,74532
0,3	0,29131	0,65009	1,3654	1,19115	2,9044	1,39686	4,7000	1,47760
0,4	0,37995	0,61912	1,7980	1,07179	3,4677	1,20130	5,3947	1,23709
0,5	0,46212	0,58220	1,9853	0,94754	3,8659	1,02326	5,8414	1,03764
0,6	0,53705	0,54105	2,2044	0,82750	4,1469	0,86947	6,1391	0,87488
0,7	0,60437	0,49735	2,3751	0,71690	4,3481	0,73928	6,3456	0,74122
0,8	0,66404	0,45267	2,5074	0,61796	4,4949	0,62955	6,4941	0,63023
0,9	0,71630	0,40831	2,6099	0,53102	4,6042	0,53690	6,6039	0,53713
1,0	0,76159	0,36533	2,6897	0,45543	4,6870	0,45837	6,6870	0,45846
1,1	0,80050	0,32453	2,7520	0,39011	4,7508	0,39155	6,7508	0,39158
1,2	0,83366	0,28655	2,8010	0,33382	4,8005	0,33454	6,8005	0,33455
1,3	0,86172	0,25141	2,8398	0,28543	4,8395	0,28578	6,8395	0,28578
1,4	0,88535	0,21954	2,8706	0,24386	4,8705	0,24404	6,8705	0,24404
1,5	0,90515	0,19087	2,8953	0,20817	4,8952	0,20826	6,8952	0,20826
1,6	0,92167	0,16528	2,9150	0,17756	4,9150	0,17760	6,9150	0,17760
1,7	0,93541	0,14263	2,9310	0,15131	4,9310	0,15133	6,9310	0,15133
1,8	0,94681	0,12271	2,9438	0,12882	4,9438	0,12884	6,9438	0,12883
1,9	0,95624	0,10527	2,9542	0,10957	4,9542	0,10958	6,9542	0,10958
2,0	0,96403	0,09009	2,9627	0,09311	4,9627	0,09311	6,9627	0,09312
3,0	0,9950	0,0150	2,9950	0,0150	4,9950	0,0150	6,9950	0,0150
4,0	0,9993	0,0028	2,9993	0,0028	4,9993	0,0028	6,9993	0,0028

Download free eBooks at bookboon.com

4.1.18 MAGNETIC REFRIGERATION SYSTEMS [1]:

With this background on the thermodynamic and magnetic properties of paramagnetic materials, we can now look into the application of adiabatic demagnetization in maintaining temperatures below 1.0 K. Such a refrigeration system has been developed by Daunt et al. (1954) at Ohio State University. A schematic of this refrigerator and the T-s diagram are shown in Fig. below. Because the working medium is a paramagnetic material (Iron ammonium alum), lines of constant magnetic field intensity appear on the temperature-entropy diagram instead of lines of constant pressure. In the ideal case, the refrigerator cycle is a Carnot cycle.

"I studied English for 16 years but... ...I finally learned to speak it in just six lessons"
Jane, Chinese architect
ENGLISH OUT THERE
Click to hear me talking before and after my unique course download

Download free eBooks at bookboon.com
Click on the ad to read more

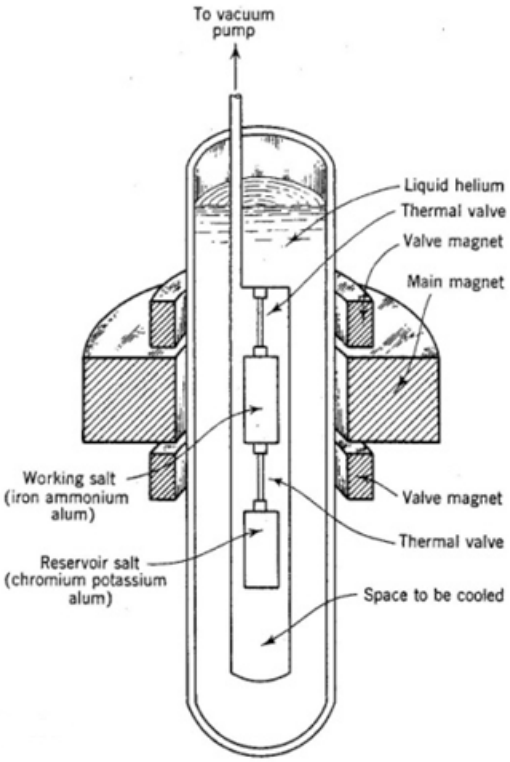


Fig. Magnetic refrigerator schematic.

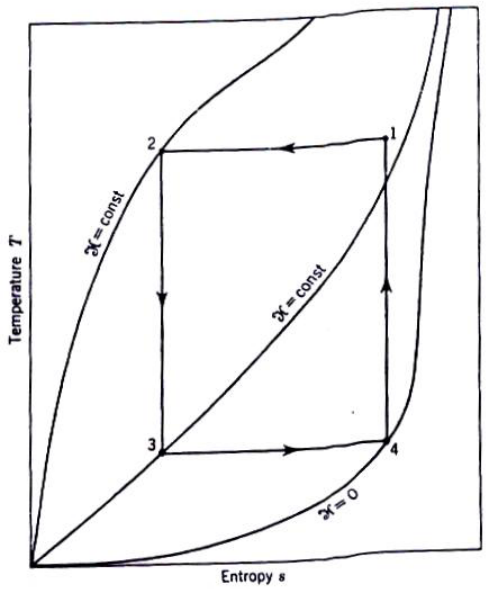


Fig. Thermodynamic cycle for the magnetic refrigerator.

Download free eBooks at bookboon.com

The sequence of operations for the magnetic refrigerator is as follows.

Process 1-2: The magnetic field is applied to the working salt while the upper thermal valve is open and the lower thermal valve is closed. When the upper thermal valve is open, heat may be transferred from the working salt to the liquid-helium bath, thereby maintaining the salt temperature fairly constant. The thermal valve between the working salt and the reservoir salt is closed so that heat will not flow back into the low-temperature reservoir during this process.

Process 2-3: Both thermal valves are closed, and the magnetic field around the working salt is reduced adiabatically to some intermediate value. During this process, tee temperature of the working salt decreases.

Process 3-4: The thermal valve between the working salt and the reservoir salt is opened, and the field around the working salt is reduced to zero while heat is absorbed isothermally by the working salt from the reservoir salt.

Process 4-1: Both thermal valves are closed, and the magnetic field around the working salt is adiabatically increased to its original value.

The energy absorbed as heat from the reservoir salt in the ideal case is given by

$$Q_a = mT_3(s_4 - s_3) \qquad (\ldots A)$$

because the process 3-4 is reversible and isothermal ideally. The entropy values may be determined from the Brillouin expression, given above. The quantity 'm' is the mass of the working salt, and the subscripts refer to the numbered points in Fig. 4.28. In the ideal case, the energy rejected as heat from the working salt is given by

$$Q_r = mT_1(s_2 - s_1) = -mT_2(s_4 - s_3)$$

Applying the First Law to the entire cycle, we find the work requirement for one cycle:

$$W_{net} = Q_a + Q_r = -m(T_1 - T_3)(s_4 - s_3) \qquad (\ldots B)$$

From eqn. (A) and (B), we see that the coefficient of performance for the ideal magnetic refrigerator is the same as that for a Carnot refrigerator.

Download free eBooks at bookboon.com

Because of the irreversibility's involved, the ideal performance of the refrigerator is not attained in practice. The actual performance of the magnetic refrigerator constructed by Daunt et al. is compared with the ideal performance in Fig. below. The mass of the working salt is 15 g, and the helium bath is maintained at a temperature of 1.11 K. The sequence of processes is carried out so that one cycle requires about 2 minutes to complete.

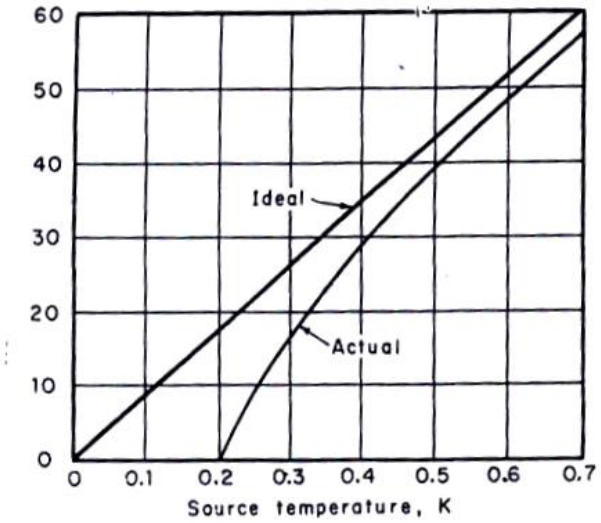
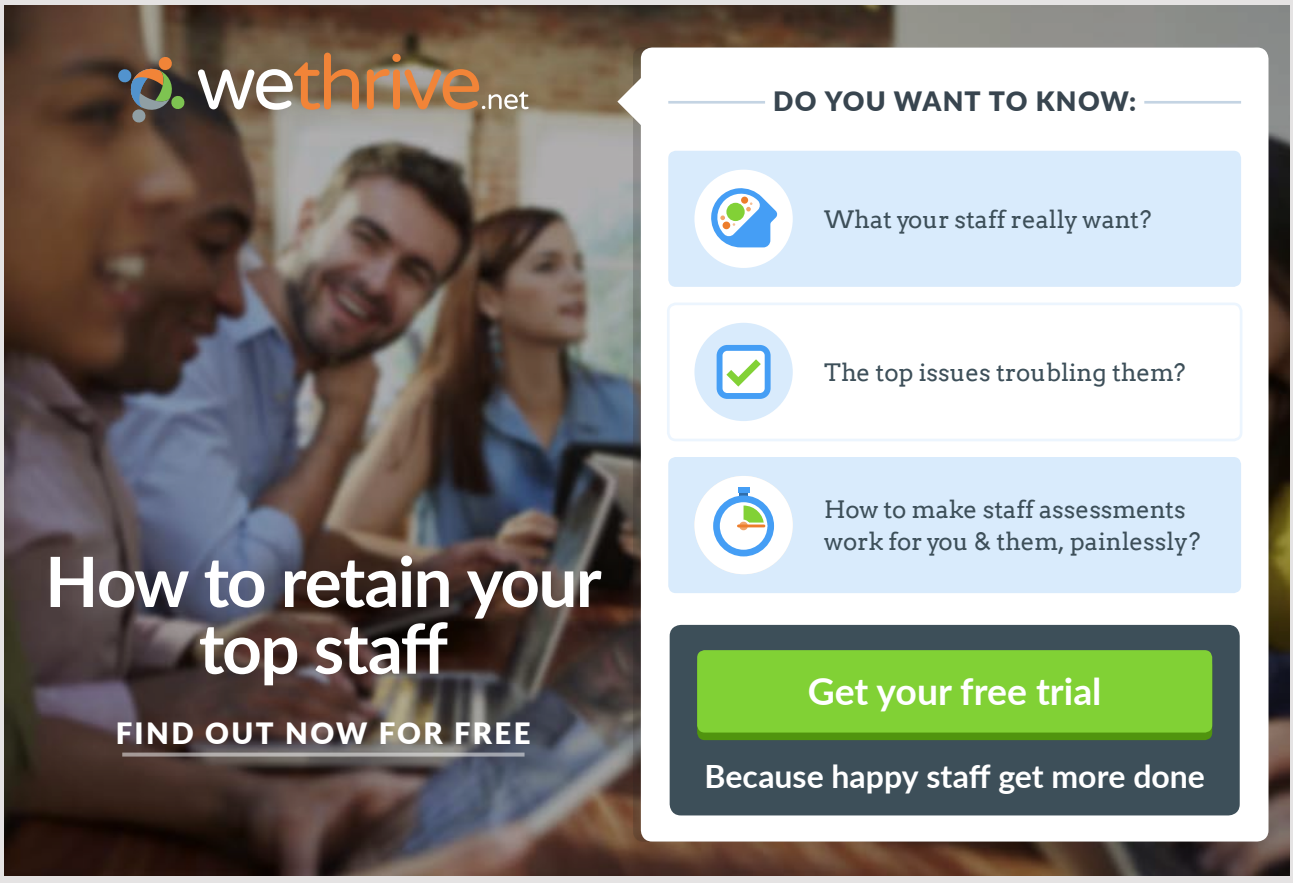


Fig. Actual and ideal performance of the magnetic refrigerator.


wethrive.net
How to retain your top staff
FIND OUT NOW FOR FREE
DO YOU WANT TO KNOW:
What your staff really want?
The top issues troubling them?
How to make staff assessments work for you & them, painlessly?
Get your free trial
Because happy staff get more done

Download free eBooks at bookboon.com
Click on the ad to read more

The working salt used in the magnetic refrigerator by Daunt et al. was an iron ammonium alum salt, and the reservoir salt (which was used as a "thermal flywheel" to smooth out temperature fluctuations in the space to be cooled) was chromium potassium alum. These salts have *low thermal conductivities*; therefore, heat transfer to and from the salts poses quite a problem. Copper fins and 3-mm (1/8-in.) lengths of fine copper wire (0.05-mm to 0.08-mm diameter) were embedded in the salt pellets to improve the heat-transfer situation. About 1 g of copper wire and 1 g of silicone vacuum grease were mixed with the 15 g of paramagnetic salt, and the pellet was formed by pressing under a pressure of 20 MPa. The vacuum grease acted as a binder and improved the mechanical stability of the salt pellet.

One of the advantages of the magnetic refrigerator is that it can operate effectively in zero gravity. Because of this characteristic, magnetic refrigerators have been used to cool infrared bolometers in space systems by NASA.

4.1.19 DILUTION REFRIGERATORS [1]:

The idea that cooling could be achieved by means of dilution of He^3 by super fluid He^4 was first suggested by H. London (1951). London presented a practical technique for the dilution refrigerator (London et al. 1962). Hall et al. (1966) and Neganov et al. (1966) constructed and operated dilution refrigerators that reached 0.065 K (0.11°R) and 0.025 K (0.045°R), respectively.

A schematic of a He^3-He^4 dilution refrigerator is shown in Fig. below:

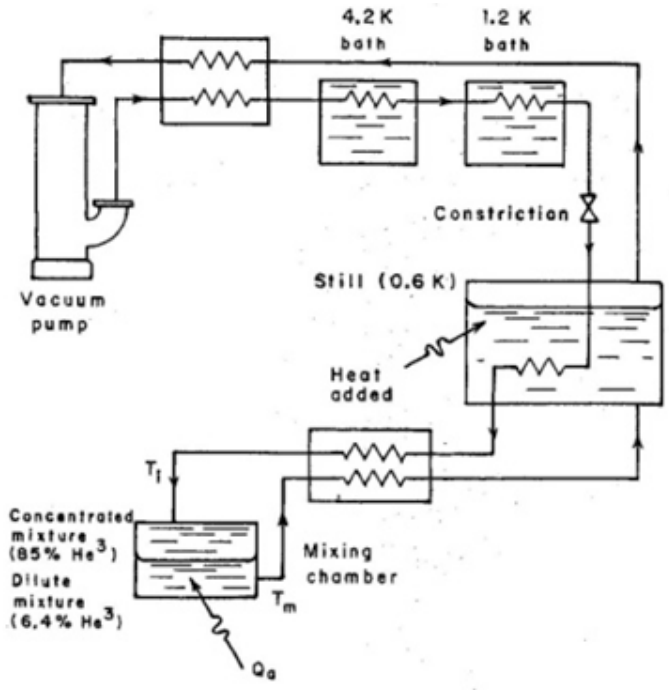


Fig. Schematic diagram of He^3-He^4 Dilution refrigerator

Download free eBooks at bookboon.com

The gas (which is practically pure He) is "compressed" in a vacuum pump from about 4 Pa (0.03 torr) to a pressure on the order of 4 kPa (30 torr), then cooled in a heat exchanger and a liquid helium bath at 4.2 K. The gas is next condensed in a bath of liquid helium boiling at about 1.2 K. The liquid He^3 expands through a constriction (capillary tube) and is cooled further in the still, which operates at about 0.6 K. The liquid is again cooled in another heat exchanger before entering the mixing chamber, where the He^3 is mixed with He^4 at temperatures between 0.005 K and 0.050 K.

In the mixing chamber, the liquid separates into two phases – a less dense concentrated He^3 mixture and a more dense dilute He^3 mixture. The temperature-composition diagram for He^3-He^4 mixtures is shown in Fig. below.

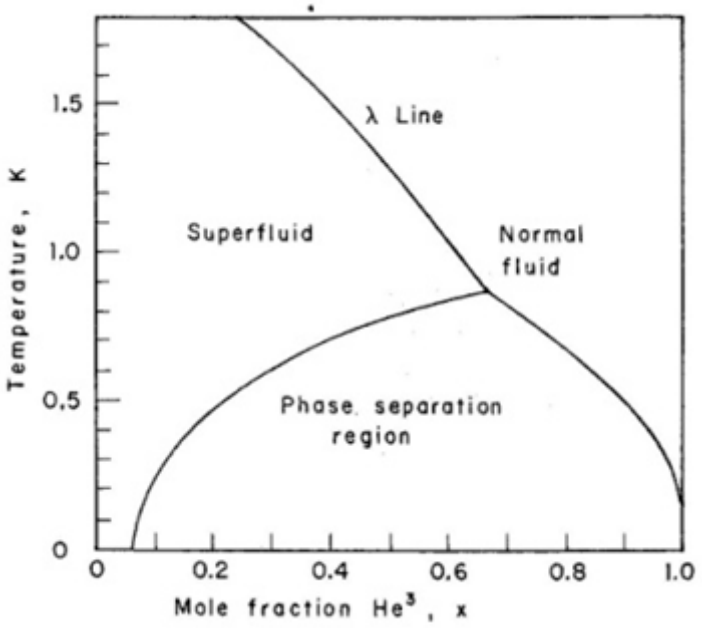


Fig. Phase diagram of He^3-He^4 mixture

Download free eBooks at bookboon.com

Table below gives numerical values:

Phase separation temperature curve for liquid He^3-He^4 mixtures (Radebaugh 1967)

Mole fraction He^3 x	Temperature (K)	Mole fraction He^3 x	Temperature (K)
0.0640	0.0	0.12	0.2902
0.0641	0.01486	0.13	0.3166
0.0642	0.0208	0.14	0.3412
0.0643	0.0253	0.15	0.3644
0.0645	0.0323	0.16	0.3863
0.0647	0.0377	0.17	0.4072
0.065	0.0445	0.18	0.4272
0.066	0.0606	0.20	0.4647
0.067	0.0721	0.25	0.5460
0.068	0.0817	0.30	0.6097
0.070	0.0970	0.35	0.6558
0.072	0.1102	0.40	0.6843
0.074	0.1223	0.847	0.60
0.076	0.1335	0.876	0.55
0.078	0.1436	0.904	0.50
0.080	0.1528	0.928	0.45
0.085	0.1747	0.949	0.40
0.090	0.1946	0.968	0.35
0.095	0.2129	0.982	0.30
0.10	0.2301	0.9965	0.20
0.11	0.2616	1.000	0.10

Download free eBooks at bookboon.com
Click on the ad to read more

The He^3 molecules "expand" from the concentrated phase into the dilute phase (actually, diffuse through the dilute phase), and the mixture temperature would tend to decrease; however, heat is added to the mixing chamber from the low temperature region to maintain a constant temperature. This process is analogous to the isothermal expansion of a gas, except no external work is done by the He^3 in the mixing chamber.

The dilute mixture returns through the heat exchanger to the still, where heat is added to evaporate the He^3 from the mixture. The concentration of He^3 in the liquid phase in the still is approximately 1.0 percent, and the composition of the vapor is around 95 to 98 percent He)[3].

The refrigeration effect of the dilution refrigerator may be determined by application of the First Law to the mixing chamber:

$$\dot{Q}_a = \dot{n}_3 \left(h_m - h_i \right) \tag{4.103}$$

where n_3 is the molar flow rate of He^3, h_m is the molar enthalpy of the He^3 in the dilute phase leaving the mixing chamber, and h, is the molar enthalpy of the practically pure He) entering the mixing chamber. Radebaugh (1967) noted that, *for temperatures below about 0.04 K, the enthalpies could be approximated by*:

$$h_m = C_1 T_m^2, \text{ where } C_1 = 94 \text{ J/mol-K}^2 \tag{4.104}$$

$$h_i = C_2 T_i^2, \text{ where } C_2 = 12 \text{ J/mol-K}^2 \tag{4.105}$$

where T_m is the temperature of the dilute phase leaving the mixing chamber, and T, is the temperature of the practically pure He^3 entering the mixing chamber.

A typical He3 flow rate for dilution refrigerators is about 1.0×10^{-4} mol/so. For an ideal heat exchanger ($T_i = T_m$) and a mixing chamber temperature of 0.040 K, the maximum refrigeration effect should be

$$\dot{Q}_a = \left(10^{-4}\right)\left(94 - 12\right)\left(0.040\right)^2 = 13.1 \times 10^{-6} \text{ W} = 13.1 \ \mu\text{W}$$

One of the more critical aspects in the design of a dilution refrigerator is the design of the heat exchanger between the mixing chamber and the still. From eqn. (4.103), we note that the temperature ratio for zero heat addition is $T_i/T_m = (94/12)^{1/2} = 2.80$. For $T_m = 0.04$ K, the largest value of T, for finite heat addition is 0.112 K, or the maximum temperature difference at the cold end of the exchanger is $(T_i - T_m) = 0.072$ K. Large surface areas per unit volume have been achieved for heat exchangers using sintered metal elements within the flow passages.

Download free eBooks at bookboon.com

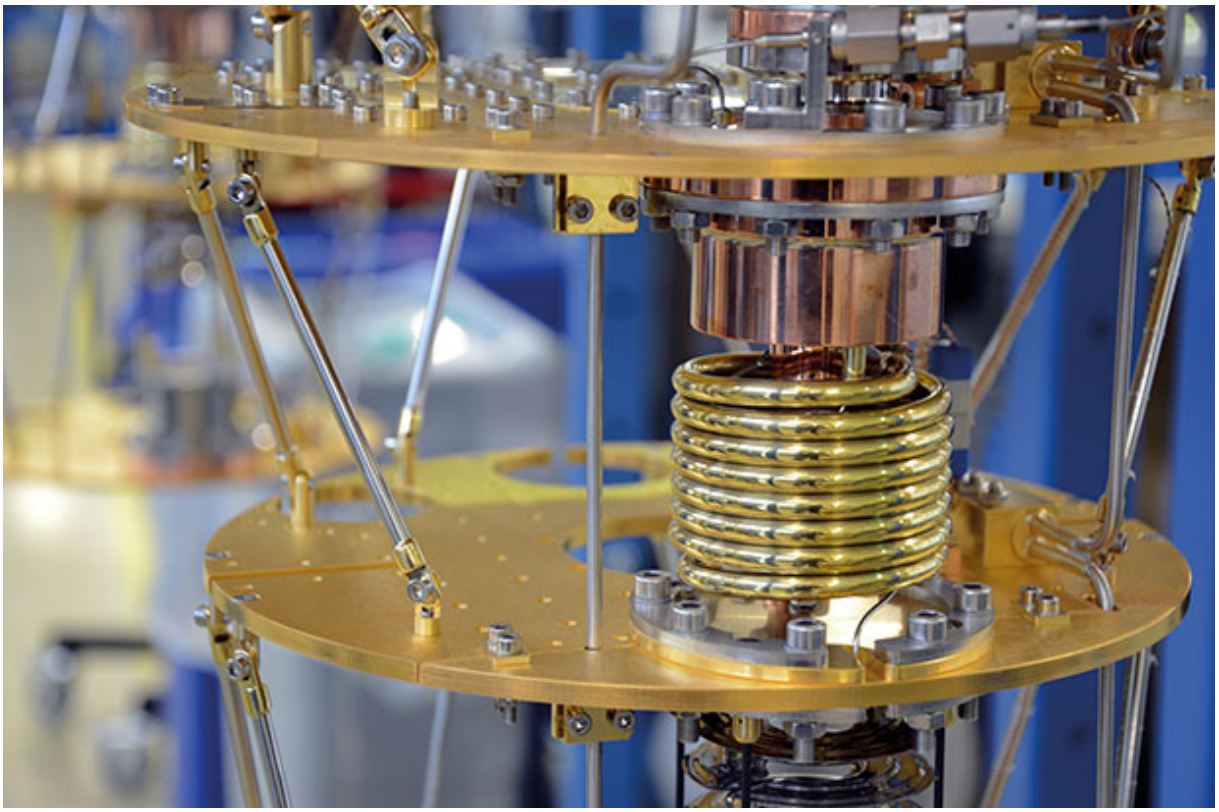
4.1.20 AN EXAMPLE OF COMMERCIAL DILUTION REFRIGERATORS:

He3-He4 dilution refrigerators produced by Oxford Instruments, U.K.:

(Ref: https://www.oxford-instruments.com/products/cryogenic-and-superconducting-magnet-research-tool/systems-for-millikelvin-temperatures)

Following is an extract from the above website:

Dilution refrigerators-continuous cooling to below 5 milliKelvin temperatures:



"Oxford Instruments' dilution refrigerators provide continuous cooling below 5 millikelvin (mK) temperatures. They are used for a wide range of research applications in quantum computing, spintronics, quantum transport, mechanical resonators, photonics, astronomy, neutron scattering and much more.

We offer traditional "wet" systems operating in a liquid helium environment as well as "dry" systems, where the cold environment is provided by a closed cycle mechanical cooler.

Both wet and dry refrigerators can be integrated with a solenoid or vector rotate superconducting magnet, a sample exchange mechanism, high-frequency signal lines, sample manipulators, windows, optical fibres, magnetic shields and many other experimental options.

Download free eBooks at bookboon.com

With over 220 systems installed worldwide, **Triton**™ is used in world-leading research across condensed matter physics, with a particular focus on advanced computing, quantum technologies, spintronics and optics."

Typical data:

	With DU7-500 dilution unit	With DU7-300 dilution unit
Base T:	8 mK typical, < 10 mK guaranteed	8 mK typical, < 10 mK guaranteed
At 100 mK:	500 μW typical, 450 μW guaranteed	300 μW typical, 250 μW guaranteed
At 20 mK:	15 μW typical, 12 μW guaranteed	10 μW typical, 8 μW guaranteed
^{3}He requirement:	18 litres STP	11 litres STP

This e-book
is made with
SetaPDF

SETASIGN

PDF components for **PHP** developers

www.setasign.com

Download free eBooks at bookboon.com

Click on the ad to read more

4.1.21 ADIABATIC NUCLEAR DEMAGNETIZATION [1, 15, 16, 17]:

One variant of adiabatic demagnetization that continues to find substantial research application is nuclear demagnetization refrigeration (NDR). NDR follows the same principles, but in this case the cooling power arises from the magnetic dipoles of the nuclei of the refrigerant atoms, rather than their electron configurations. Since these dipoles are of much smaller magnitude, they are less prone to self-alignment and have lower intrinsic minimum fields. This allows NDR to cool the nuclear spin system to very low temperatures, often 1 μK or below. Unfortunately, the small magnitudes of nuclear magnetic dipoles also makes them less inclined to align to external fields. Magnetic fields of 3 Teslas or greater are often needed for the initial magnetization step of NDR.

In NDR systems, the initial heat sink must have a very low temperature (10–100 mK). This pre-cooling is often provided by the mixing chamber of a dilution refrigerator or a paramagnetic salt. [31]

Schematic diagram of a NDR is shown below [1]. It is a two stage apparatus: upper stage consists of an ordinary paramagnetic salt (ex: chromium potassium alum) to cool to about 0.0056 K and the lower stage consists of a nuclear paramagnetic material (generally, copper).

Working of NDR is as follows:

Process 1-2: Both the nuclear and electronic paramagnetic stages are placed in a high field, while the heat of magnetization is carried away by the helium exchange gas to a LHe bath.

Process 2-3: Both the stages are isolated from the LHe bath by closing the thermal valve and pumping out the helium exchange gas. The two stages are still in thermal contact with each other. Magnetic field is removed from the ordinary paramagnetic salt but the nuclear stage is still in the high magnetic field. The temp of upper stage drops due to adiabatic demagnetization, and there is a heat transfer from nuclear stage to the electronic stage. Both stages finally reach a temp of the order of 0.0056 K (i.e. approx. 5 mK).

Process 3-4: Thermal contact between the two stages is broken by closing a thermal valve and the magnetic field is removed from the nuclear stage. To maintain the nuclear magnetic entropy constant, temp of nuclear stage drops to a very low value.

Download free eBooks at bookboon.com

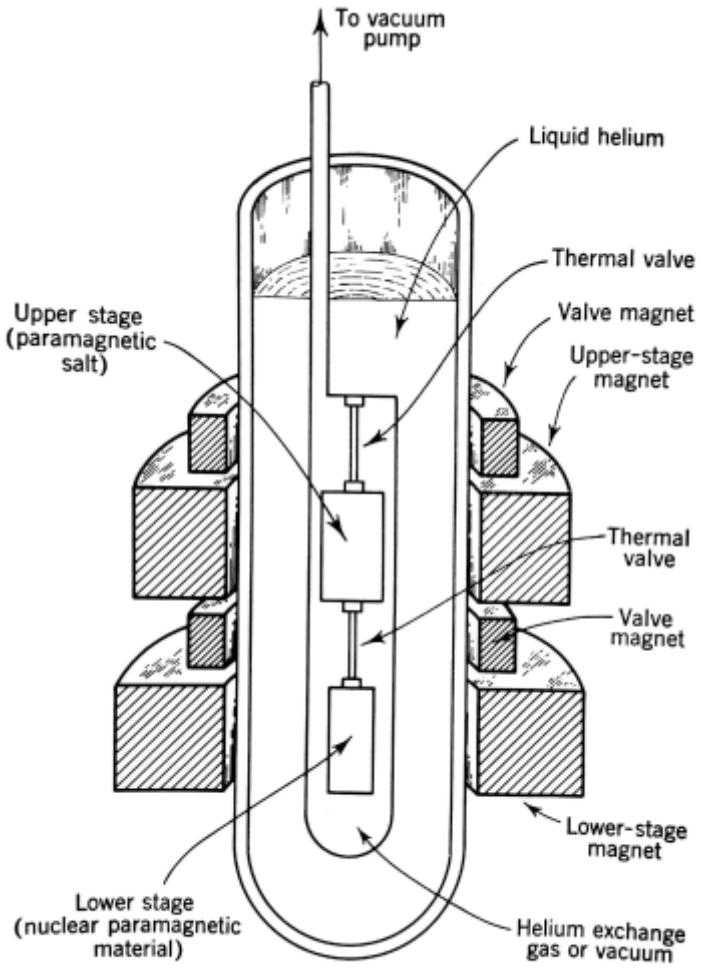


Fig. Nuclear-demagnetization apparatus.

High magnetic fields required are generally obtained using Superconducting magnets [16, 17]. Some data for the superconducting magnets used are given below:

Cryomagnetics [16] have manufactured NbTi Nuclear Demagnetization Magnets up to 9T (4.2K), 11T (2.2K). And, Nb3Sn based versions delivered include up to 17T (4.2K), 19T (2.2K).

American Magnetics Inc. website [17] states as follows: "Magnets for nuclear demagnetization are required to have a large volume at high fields into which the sample to be demagnetized (frequently a bundle of copper wires) is placed. A He^3-He^4 dilution refrigerator is generally used to cool the sample while it is in the magnetic field and before it is demagnetized. During the demagnetization process, the refrigerator is thermally decoupled from the sample by means of a superconductive heat switch.

Download free eBooks at bookboon.com

To accomplish these operations, the high field magnet used to magnetize the copper wire bundle must be compensated so it will not affect the operation of the heat switch or switches and so it will not generate appreciable eddy currents in the mixing chamber of the dilution refrigerator. This field free region is also used for the ultra low temperature experiment itself.

The magnet system for this application is a high field magnet, a nulling coil, and a series of compensating coils mounted on an integral cylinder extending above the main magnet. The dilution refrigerator and the experiment itself are located inside the compensation coils where the field is typically reduced to less than 3 mT when the main field is at 8 T. In some cases, an additional highly homogeneous magnet is placed in the compensated region to permit the *temperature to be measured using nuclear magnetic resonance techniques*.

The dilution refrigerator and magnet designs must be closely coordinated to assure that the proper low magnetic fields are achieved at the superconducting heat switches and that adequate space is available inside the magnet for the various refrigerator components."

Deloitte.

© Deloitte & Touche LLP and affiliated entities.

Discover the truth at www.deloitte.ca/careers

Download free eBooks at bookboon.com

Click on the ad to read more

4.2 PROBLEMS SOLVED WITH MATHCAD:

Note: In this chapter, Mathcad functions written earlier in the free ebook, viz. "Cryogenic Engineering: Software Solutions –Vol II B" are used to get properties of cryogenic fluids.

(Ref: http://bookboon.com/en/cryogenic-engineering-software-solutions-vol-ii-b-ebook)

Prob. 4.2.1 Write a Mathcad Function to calculate the COP and Work requirement for a Thermodynamically Ideal, Isothermal source system.[1]

Note: Temps in following Function must be in Kelvin.

$$\text{COP\_ideal\_isoth}(Tc, Th) := \left| \begin{array}{l} \text{COP}_i \leftarrow \dfrac{Tc}{Th - Tc} \\ \text{WbyQ} \leftarrow \dfrac{1}{\text{COP}_i} \\ \begin{pmatrix} \text{"Tc(K)"} & \text{"Th(K)"} & \text{"COP\_ideal"} & \text{"(W/Q) = 1/COP\_ideal"} \\ Tc & Th & \text{COP}_i & \text{WbyQ} \end{pmatrix} \end{array} \right.$$

Ex: $Th := 300\ K \qquad Tc := 77\ K$

$$\text{COP\_ideal\_isoth}(Tc, Th) = \begin{pmatrix} \text{"Tc(K)"} & \text{"Th(K)"} & \text{"COP\_ideal"} & \text{"(W/Q) = 1/COP\_ideal"} \\ 77 & 300 & 0.345 & 2.896 \end{pmatrix}$$

$$\text{COP\_id}(Tc, Th) := \text{COP\_ideal\_isoth}(Tc, Th)_{1,2} \qquad \text{COP\_id}(Tc, Th) = 0.345$$

$$\text{WbyQ}(Tc, Th) := \text{COP\_ideal\_isoth}(Tc, Th)_{1,3} \qquad \text{WbyQ}(Tc, Th) = 2.896$$

Values of COP and WbyQ for some important temps, with the sink at 300 K:

$Tc := (0.001\ \ 0.0056\ \ 0.01\ \ 0.278\ \ 4.2\ \ 20.4\ \ 77.2\ \ 112)$ .K..vector of Tc values

$k := 0..7$ ...range variable for k, the index

Download free eBooks at bookboon.com

Then, we get:

$Tc_{0,k}$ =	$COP\_id(Tc_{0,k}, Th)$	$WbyQ(Tc_{0,k}, Th)$
$1 \cdot 10^{-3}$	$3.333 \cdot 10^{-6}$	$3 \cdot 10^{5}$
$5.6 \cdot 10^{-3}$	$1.867 \cdot 10^{-5}$	$5.357 \cdot 10^{4}$
0.01	$3.333 \cdot 10^{-5}$	$3 \cdot 10^{4}$
0.278	$9.275 \cdot 10^{-4}$	$1.078 \cdot 10^{3}$
4.2	0.014	70.429
20.4	0.073	13.706
77.2	0.346	2.886
112	0.596	1.679

Plot COP vs Tc, with the sink at 300 K:

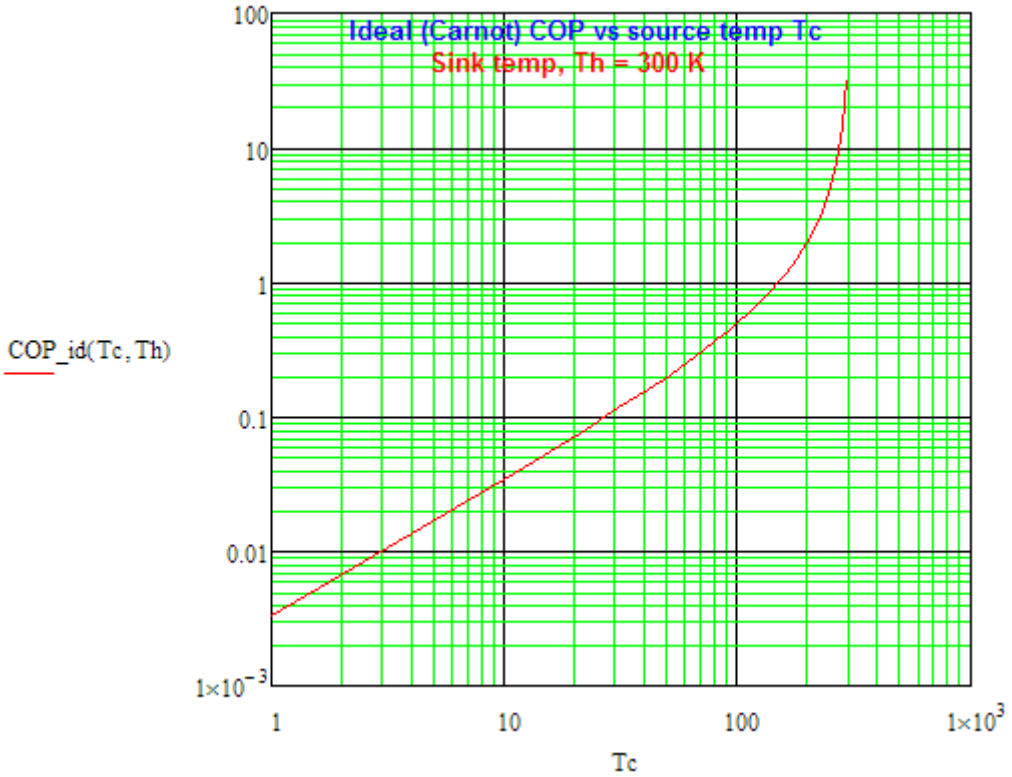
$Th := 300$ K .... sink temp $Tc := 4.2$ K

$COP\_ideal\_isoth(Tc, Th) =$

"Tc(K)"	"Th(K)"	"COP_ideal"	"(W/Q) = 1/COP_ideal"
4.2	300	0.014	70.429

$Tc := 1, 11 .. 301$ K .... define a range variable

Plot:



Download free eBooks at bookboon.com

Prob. 4.2.2 A Carnot refrigerator operates between 300K and 72 K. Determine the COP [1]

Data: $Th := 300\ K$ $Tc := 72\ K$

Then, we have:

$$COP\_ideal\_isoth(Tc, Th) = \begin{pmatrix} "Tc(K)" & "Th(K)" & "COP\_ideal" & "(W/Q) = 1/COP\_ideal" \\ 72 & 300 & 0.316 & 3.167 \end{pmatrix}$$

Therefore: $COP = 0.316$ **...Ans.**

==========

Prob. 4.2.3 Determine the minimum work required to remove 200 W from a region at 20.4 K if the sink temp is: (a) 300 K (b) 77 K. [1]

Minimum work is given for a Carnot refrigerator.

be
your degree
© 2013 Accenture. All rights reserved.
Bring your talent and passion to a global organization at the forefront of business, technology and innovation. Discover how great you can be.
Visit accenture.com/bookboon
Be greater than.
consulting | technology | outsourcing
accenture
High performance. Delivered.
Download free eBooks at bookboon.com
Click on the ad to read more

Therefore:

Data:

(a) $Th := 300$ K $Tc := 20.4$ K $Q := 200$ W

$$COP\_ideal\_isoth(Tc, Th) = \begin{pmatrix} \text{"Tc(K)"} & \text{"Th(K)"} & \text{"COP\_ideal"} & \text{"(W/Q) = 1/COP\_ideal"} \\ 20.4 & 300 & 0.073 & 13.706 \end{pmatrix}$$

We see that: $WbyQ := 13.706$ W/W

Then, to remove 200 W, work required:

$W := Q \cdot WbyQ$ i.e. $W = 2.741 \times 10^3$ **W ..... Ans.**

Data:

(b) $Th := 77$ K $Tc := 20.4$ K $Q := 200$ W

$$COP\_ideal\_isoth(Tc, Th) = \begin{pmatrix} \text{"Tc(K)"} & \text{"Th(K)"} & \text{"COP\_ideal"} & \text{"(W/Q) = 1/COP\_ideal"} \\ 20.4 & 77 & 0.36 & 2.775 \end{pmatrix}$$

We see that: $WbyQ := 2.775$ W/W

Then, to remove 200 W, work required:

$W := Q \cdot WbyQ$ i.e. $W = 555$ **W ..... Ans.**

==========

Prob. 4.2.4 Write a Mathcad Function to calculate COP and WbyQ for a Thermodynamically Ideal, Isobaric source system.

Note: Temps in following Function must be in Kelvin.

For a 'real gas' N2:

$$COP\_ideal\_isobaric\_N2(P1, T1, T2, T0) := \left| \begin{array}{l} h1 \leftarrow enthalpy\_N2(P1, T1) \\ s1 \leftarrow entropy\_N2(P1, T1) \\ h2 \leftarrow enthalpy\_N2(P1, T2) \\ s2 \leftarrow entropy\_N2(P1, T2) \\ COP_i \leftarrow \dfrac{h2 - h1}{T0 \cdot (s2 - s1) - (h2 - h1)} \\ WbyQ \leftarrow \dfrac{1}{COP_i} \\ \begin{pmatrix} \text{"P1(bar)"} & \text{"T1 (K)"} & \text{"T2 (K)"} & \text{"T0(K)"} & \text{"COP\_ideal"} & \text{"(W/Q) = 1/COP\_ideal"} \\ P1 & T1 & T2 & T0 & COP_i & WbyQ \end{pmatrix} \end{array} \right.$$

Download free eBooks at bookboon.com

Ex: $P1 := 10 \quad bar \qquad P2 := P1$

$T1 := 112 \quad K \qquad T2 := 190 \quad K \qquad T0 := 300 \quad K$

Then, using the Function written above:

$$COP\_ideal\_isobaric\_N2(P1, T1, T2, T0) = \begin{pmatrix} \text{"P1(bar)"} & \text{"T1 (K)"} & \text{"T2 (K)"} & \text{"T0(K)"} & \text{"COP\_ideal"} & \text{"(W/Q) = 1/COP\_ideal"} \\ 10 & 112 & 190 & 300 & 0.946 & 1.057 \end{pmatrix}$$

Prob. 4.2.5 Write a Mathcad Function to calculate COP and WbyQ for a Thermodynamically Ideal, Isobaric source system.

For an Ideal gas:

$$COP\_ideal\_isobaric\_IdealGas(T1, T2, T0) := \left| \begin{array}{l} COP_i \leftarrow \dfrac{\dfrac{T2}{T1} - 1}{\dfrac{T0}{T1} \cdot \ln\left(\dfrac{T2}{T1}\right) - \dfrac{T2}{T1} + 1} \\ WbyQ \leftarrow \dfrac{1}{COP_i} \\ \begin{pmatrix} \text{"T1 (K)"} & \text{"T2 (K)"} & \text{"T0 (K)"} & \text{"COP\_ideal"} & \text{"(W/Q) = 1/COP\_ideal"} \\ T1 & T2 & T0 & COP_i & WbyQ \end{pmatrix} \end{array} \right.$$

Ex: $P1 := 10 \quad bar$

$T1 := 112 \quad K \qquad T2 := 190 \quad K \qquad T0 := 300 \quad K$

Using the Function written above, we get:

$$COP\_ideal\_isobaric\_IdealGas(T1, T2, T0) = \begin{pmatrix} \text{"T1 (K)"} & \text{"T2 (K)"} & \text{"T0 (K)"} & \text{"COP\_ideal"} & \text{"(W/Q) = 1/COP\_ideal"} \\ 112 & 190 & 300 & 0.968 & 1.033 \end{pmatrix}$$

Thus, for an ideal gas, we get: COP_i = 0.968.

Download free eBooks at bookboon.com

Note that in the Prob.4.2.4, for the same values of T1, T2 and T0, the COP_ideal was 0.946 treating Nitrogen as a 'real gas'.

Let A = (T1/T0), and B = (T2/T1)

Then, we re-write the above eqn as follows:

$$COP\_i(A,B) := \frac{B - 1}{\left(\frac{1}{A}\right)\cdot \ln(B) - B + 1}$$

Now:

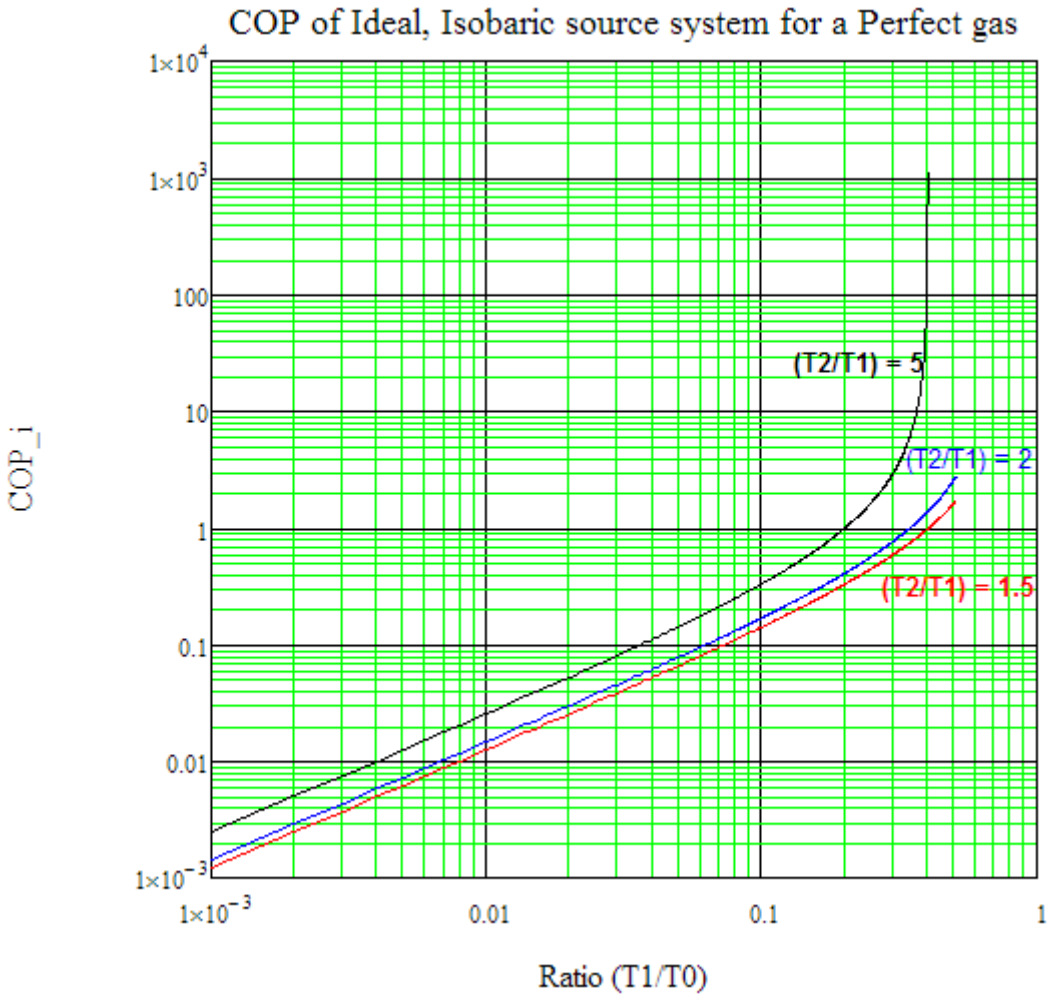
A := 0.001, 0.002.. 0.51 ...define a range variable

The Wake
the only emission we want to leave behind
Low-speed Engines Medium-speed Engines Turbochargers Propellers Propulsion Packages PrimeServ
The design of eco-friendly marine power and propulsion solutions is crucial for MAN Diesel & Turbo.
Power competencies are offered with the world's largest engine programme – having outputs spanning
from 450 to 87,220 kW per engine. Get up front!
Find out more at www.mandieselturbo.com
Engineering the Future – since 1758.
MAN Diesel & Turbo
MAN

Download free eBooks at bookboon.com

Click on the ad to read more

Plot:



Prob. 4.2.6 An ideal, isobaric source refrigerator operates with 300 K as the sink temp, while the source temp varies between 70 K and 90 K. For an ideal gas, determine the COP. Compare this with the COP of a Carnot refrigerator operating between: (a) 300 K and 70 K (b) 300 K and 90 K. [1]

Data:

$T1 := 70 \quad K \qquad T2 := 90 \quad K \qquad T0 := 300 \quad K$

We get:

$$COP\_ideal\_isobaric\_IdealGas(T1, T2, T0) = \begin{pmatrix} \text{"T1 (K)"} & \text{"T2 (K)"} & \text{"T0 (K)"} & \text{"COP\_ideal"} & \text{"(W/Q) = 1/COP\_ideal"} \\ 70 & 90 & 300 & 0.361 & 2.77 \end{pmatrix}$$

Download free eBooks at bookboon.com

Therefore: $COP = 0.361$ **...Ans.**

(a) For a Carnot refrig. operating between 300 K and 70 K:

$$COP\_ideal\_isoth(T1, T0) = \begin{pmatrix} "Tc(K)" & "Th(K)" & "COP\_ideal" & "(W/Q) = 1/COP\_ideal" \\ 70 & 300 & 0.304 & 3.286 \end{pmatrix}$$

Then: $COP = 0.304$ **....Ans.**

(b) For a Carnot refrig. operating between 300 K and 90 K:

$$COP\_ideal\_isoth(T2, T0) = \begin{pmatrix} "Tc(K)" & "Th(K)" & "COP\_ideal" & "(W/Q) = 1/COP\_ideal" \\ 90 & 300 & 0.429 & 2.333 \end{pmatrix}$$

Then: $COP = 0.429$ **....Ans.**

==========

Prob. 4.2.7 Determine the work requirement for an ideal, isobaric source refrigerator using an ideal gas as the working fluid that removes 2 kW of energy from the low temp source. The source temp varies from 5 K to 20 K and the sink temp is 300 K. [1]

Data:

$T1 := 5$ K $T2 := 20$ K $T0 := 300$ K $Q := 2$ kW

We get:

$$COP\_ideal\_isobaric\_IdealGas(T1, T2, T0) = \begin{pmatrix} "T1\ (K)" & "T2\ (K)" & "T0\ (K)" & "COP\_ideal" & "(W/Q) = 1/COP\_ideal" \\ 5 & 20 & 300 & 0.037 & 26.726 \end{pmatrix}$$

Therefore: $COP = 0.037$ **...Ans.**

And: $WbyQ := 26.726$ kW/kW

Therefore: $W := WbyQ \cdot Q$

i.e. $W = 53.452$ **kW .... Ans.**

==========

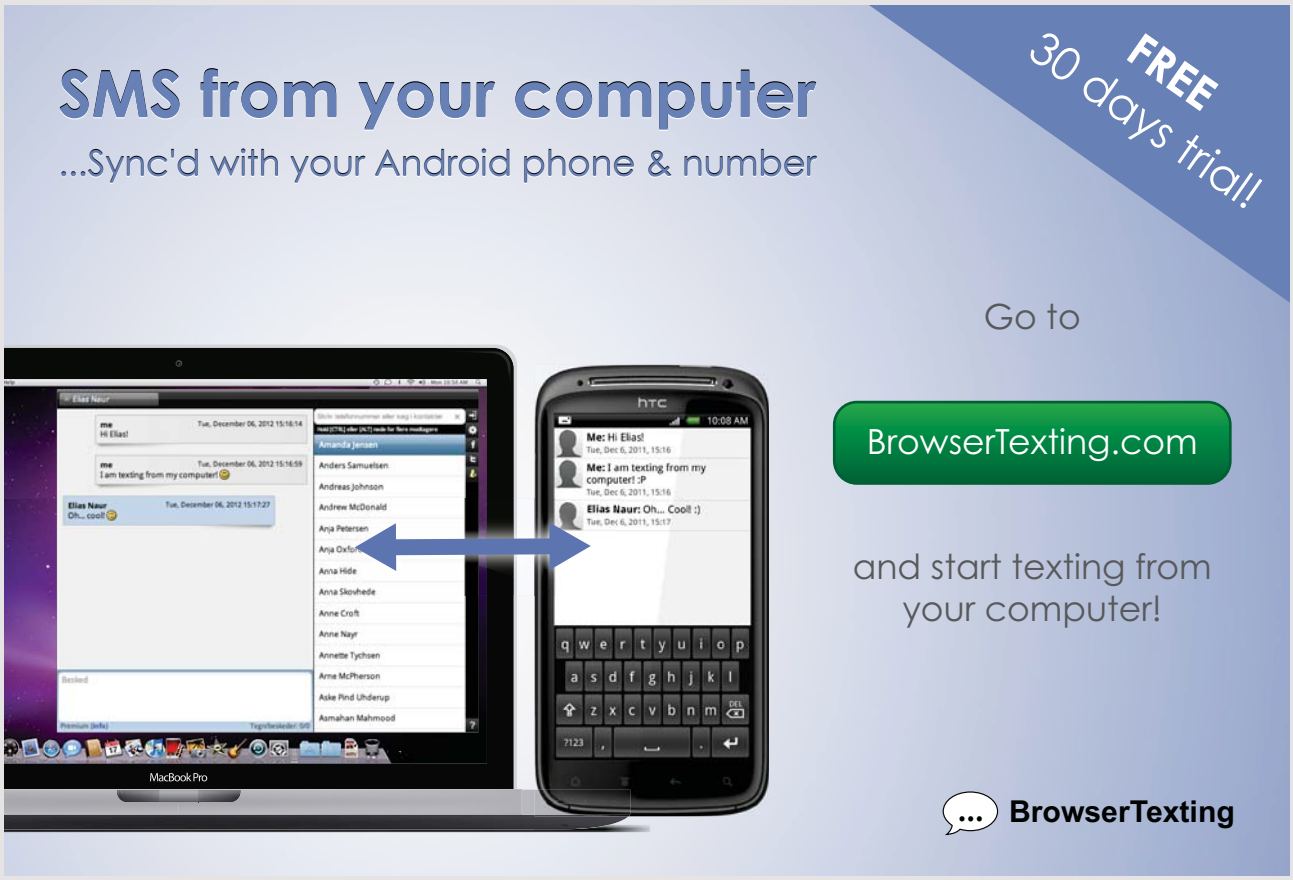
Download free eBooks at bookboon.com

Prob. 4.2.8 Write a Mathcad Function to find the parameters of a J-T Refrigeration system with Nitrogen as working fluid.

Note: Temps. in following Function must be in Kelvin, Pressures in bar.

For a 'real gas' N2:

JT_Refrign_system_N2(P1, T1, P2, eta_co, eta_HX) :=



Download free eBooks at bookboon.com
Click on the ad to read more

$h1 \leftarrow enthalpy\_N2(P1, T1)$

$s1 \leftarrow entropy\_N2(P1, T1)$

$h2 \leftarrow enthalpy\_N2(P2, T1)$

$s2 \leftarrow entropy\_N2(P2, T1)$

$hg \leftarrow HGSATP(P1)$

$$W\_by\_m \leftarrow \frac{T1 \cdot (s1 - s2) - (h1 - h2)}{eta\_co}$$

$$Qa\_by\_m \leftarrow (h1 - h2) - (1 - eta\_HX) \cdot (h1 - hg)$$

$$COP \leftarrow \frac{Qa\_by\_m}{W\_by\_m}$$

$Tsat\_N2 \leftarrow TSAT(P1)$

$$COP_i \leftarrow \frac{Tsat\_N2}{T1 - Tsat\_N2}$$

$$FOM \leftarrow \frac{COP}{COP_i}$$

"P1(bar)"	"T1 (K)"	"Tsat_N2 at P1 (K)"	"P2 (bar)"	"Qa_by_m(kJ/kg)"	"W_by_m(kJ/kg)"	"COP"	"COP_i"	"FOM"
P1	T1	Tsat_N2	P2	Qa_by_m	W_by_m	COP	COP_i	FOM

In the above program, the first line is the LHS of the Function, and defines the function. Parameters inside the brackets are Inputs with Pressures in bar, temps in Kelvin, and eta_co is the overall effcy of compressor and eta_HX is the effectiveness of the heat exchanger.

Output is given as a compact matrix in the last line of the Function. In the output, we have: P1, T1, Sat. temp of N2 at P1, P2, Qa/m in kJ/kg, Wbym in kJ/kg, COP, COP_ideal and FOM (= COP/COP_ideal).

Prob. 4.2.9 In a J-T Refrigeration system with Nitrogen as working fluid, the system works between 1 atm, 294 K and 100 atm, with isothermal compression. Efficiencies of compressor and heat exchangers are 100% each. Find out the refrig. effect, work required, COP and FOM.

Data: P1 := 1.013 bar T1 := 294 K P2 := 101.3 bar

eta_co := 1 ....overall effcy of compressor

eta_HX := 1 ...effetiveness of HX

Download free eBooks at bookboon.com

Using the Function written above, we get:

JT_Refrign_system_N2(P1, T1, P2, eta_co, eta_HX) =

"P1(bar)"	"T1 (K)"	"Tsat_N2 at P1 (K)"	"P2 (bar)"	"Qa_by_m(kJ/kg)"	"W_by_m(kJ/kg)"	"COP"	"COP_i"	"FOM"
1.013	294	77.166	101.3	20.43	401.454	0.051	0.356	0.143

Thus:

Refrig. effect: Qa_by_m = 20.43 **kJ/kg .... Ans.**

Work required: W_by_m = 401.454 **kJ/kg .... Ans.**

COP: COP = 0.051 **...Ans.**

COP_ideal: COP_i = 0.356 **... Carnot effcy .... Ans.**

FOM: FOM = 0.143 **...Ans.**

==========

Prob. 4.2.10 In a J-T Refrigeration system with Nitrogen as working fluid, the system works between 0.5 bar, 300 K and 152 bar, with isothermal compression. Efficiencies of compressor and heat exchangers are 100%. Find out all the refrig. effect, work required, COP and FOM.

Data:

P1 := 0.5 bar T1 := 300 K P2 := 152 bar TSAT(P1) = 70.889 K

eta_co := 1 ....overall effcy of compressor

eta_HX := 1 ...effetiveness of HX

Using the Function written above, we get:

JT_Refrign_system_N2(P1, T1, P2, eta_co, eta_HX) =

"P1(bar)"	"T1 (K)"	"Tsat_N2 at P1 (K)"	"P2 (bar)"	"Qa_by_m(kJ/kg)"	"W_by_m(kJ/kg)"	"COP"	"COP_i"	"FOM"
0.5	300	70.889	152	26.916	508.821	0.053	0.309	0.171

Download free eBooks at bookboon.com

Thus:

Refrig. effect: Qa_by_m = 26.916 **kJ/kg .... Ans.**

Work required: W_by_m = 508.821 **kJ/kg .... Ans.**

COP: COP = 0.053 **...Ans.**

COP_ideal: COP_i = 0.309 **... Carnot effcy .... Ans.**

FOM: FOM = 0.171 **...Ans.**

Prob. 4.2.11 Solve prob. 4.2.10, if the heat exchanger has an effectiveness of 0.965 and the efficiency of the compressor is 70%. Rest of the conditions remain the same.

Data:

P1 := 0.5 bar T1 := 300 K P2 := 152 bar TSAT(P1) = 70.889 K

eta_co := 0.7 ....overall effcy of compressor

eta_HX := 0.965 ...effetiveness of HX

Download free eBooks at bookboon.com
Click on the ad to read more

Using the Function written above, we get:

JT_Refrign_system_N2(P1, T1, P2, eta_co, eta_HX) =

"P1(bar)"	"T1 (K)"	"Tsat_N2 at P1 (K)"	"P2 (bar)"	"Qa_by_m(kJ/kg)"	"W_by_m(kJ/kg)"	"COP"	"COP_i"	"FOM"
0.5	300	70.889	152	18.531	726.887	0.025	0.309	0.082

Thus:

Refrig. effect: Qa_by_m = 18.531 **kJ/kg .... Ans.**

Work required: W_by_m = 726.887 **kJ/kg .... Ans.**

COP: COP = 0.025 **...Ans.**

COP_ideal: COP_i = 0.309 **... Carnot effcy .... Ans.**

FOM: FOM = 0.082 **...Ans.**

Plot COP against compressor effcy, as the compressor effcy varies from 0.6 to 1, for eta_HX values of 0.96, 0.98 and 1:

First, write COP as function of eta_co and eta_HX:

$$COP(eta\_co, eta\_HX) := JT\_Refrign\_system\_N2(P1, T1, P2, eta\_co, eta\_HX)_{1,6}$$

Now:

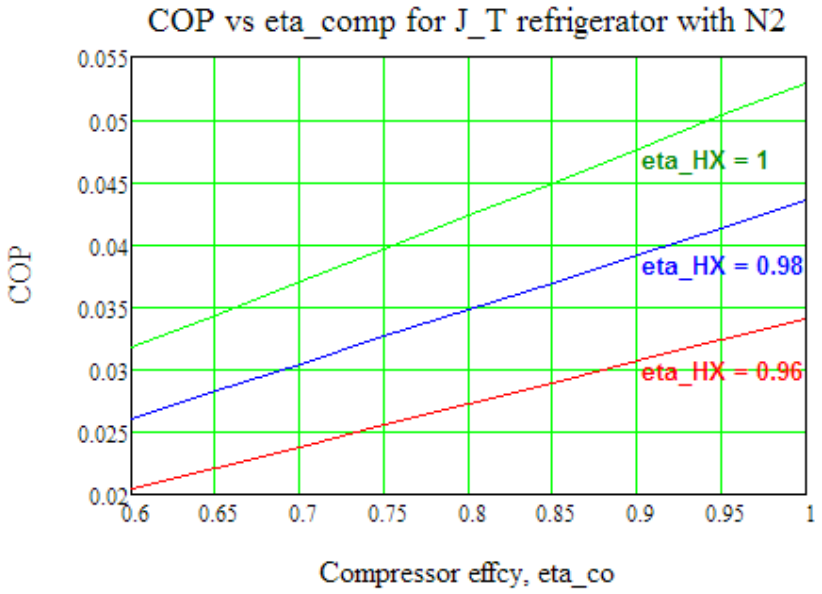
eta_co := 0.6, 0.65 .. 1 ...define a range value

And, we get:

eta_co =	COP(eta_co, 0.96)	COP(eta_co, 0.98)	COP(eta_co, 1)
0.6	0.0204	0.0261	0.0317
0.65	0.0221	0.0283	0.0344
0.7	0.0238	0.0304	0.037
0.75	0.0255	0.0326	0.0397
0.8	0.0273	0.0348	0.0423
0.85	0.029	0.037	0.045
0.9	0.0307	0.0391	0.0476
0.95	0.0324	0.0413	0.0503
1	0.0341	0.0435	0.0529

Download free eBooks at bookboon.com

Now, plot:



Prob.4.2.12 Write a Mathcad Function to find the parameters of a J-T Refrigeration system with Argon as working fluid.

Note: Temps. in the following Function must be in Kelvin, Pressures in bar.

For a 'real gas' Argon:

JT_Refrign_system_Ar(P1, T1, P2, eta_co, eta_HX) :=

h1 ← enthalpy_Ar(P1, T1)

s1 ← entropy_Ar(P1, T1)

h2 ← enthalpy_Ar(P2, T1)

s2 ← entropy_Ar(P2, T1)

hg ← HGSATP(P1)

$$W\_by\_m \leftarrow \frac{T1\cdot(s1 - s2) - (h1 - h2)}{eta\_co}$$

$$Qa\_by\_m \leftarrow (h1 - h2) - (1 - eta\_HX)\cdot(h1 - hg)$$

$$COP \leftarrow \frac{Qa\_by\_m}{W\_by\_m}$$

Tsat_Ar ← TSAT(P1)

$$COP_i \leftarrow \frac{Tsat\_Ar}{T1 - Tsat\_Ar}$$

$$FOM \leftarrow \frac{COP}{COP_i}$$

Download free eBooks at bookboon.com

"P1(bar)"	"T1 (K)"	"Tsat_Ar at P1 (K)"	"P2 (bar)"	"Qa_by_m(kJ/kg)"	"W_by_m(kJ/kg)"	"COP"	"COP_i"	"FOM"
P1	T1	Tsat_Ar	P2	Qa_by_m	W_by_m	COP	COP_i	FOM

Prob. 4.2.13 In a J-T Refrigeration system with Argon as working fluid, the system works between 1 atm, 300 K and 200 atm, with isothermal compression. Efficiencies of compressor and heat exchangers are 100%. Find out the refrig. effect, work required, COP and FOM.

Data:

P1 := 1.013 bar T1 := 300 K P2 := 202.6 bar

eta_co := 1 ....overall effcy of compressor

eta_HX := 1 ...effetiveness of HX

Using the Mathcad Function written above for Argon, w get:

JT_Refrign_system_Ar(P1, T1, P2, eta_co, eta_HX) =

"P1(bar)"	"T1 (K)"	"Tsat_Ar at P1 (K)"	"P2 (bar)"	"Qa_by_m(kJ/kg)"	"W_by_m(kJ/kg)"	"COP"	"COP_i"	"FOM"
1.013	300	87.289	202.6	33.042	325.824	0.101	0.41	0.247

Brain power
By 2020, wind could provide one-tenth of our planet's electricity needs. Already today, SKF's innovative know-how is crucial to running a large proportion of the world's wind turbines.
Up to 25 % of the generating costs relate to maintenance. These can be reduced dramatically thanks to our systems for on-line condition monitoring and automatic lubrication. We help make it more economical to create cleaner, cheaper energy out of thin air.
By sharing our experience, expertise, and creativity, industries can boost performance beyond expectations.
Therefore we need the best employees who can meet this challenge!
The Power of Knowledge Engineering
Plug into The Power of Knowledge Engineering.
Visit us at www.skf.com/knowledge
SKF

Download free eBooks at bookboon.com

Click on the ad to read more

Thus:

Refrig. effect: Qa_by_m = 33.042 **kJ/kg .... Ans.**

Work required: W_by_m = 325.824 **kJ/kg .... Ans.**

COP: COP = 0.101 **...Ans.**

COP_ideal: COP_i = 0.41 **... Carnot effcy .... Ans.**

FOM: FOM = 0.247 **...Ans.**

==========

Prob. 4.2.14 In Prob.4.2.13, if the compressor effcy = 72%, and the heat exchanger effectiveness = 98%, find out the refrig. effect, work required, COP and FOM. Other conditions remain the same.

Data:

P1 := 1.013 bar T1 := 300 K P2 := 202.6 bar

eta_co := 0.72 ....overall effcy of compressor

eta_HX := 0.98 ...effetiveness of HX

Using the Function, we get:

JT_Refrign_system_Ar(P1, T1, P2, eta_co, eta_HX) =

"P1(bar)"	"T1 (K)"	"Tsat_Ar at P1 (K)"	"P2 (bar)"	"Qa_by_m(kJ/kg)"	"W_by_m(kJ/kg)"	"COP"	"COP_i"	"FOM"
1.013	300	87.289	202.6	30.797	452.534	0.068	0.41	0.166

Thus:

Refrig. effect: Qa_by_m = 30.797 **kJ/kg .... Ans.**

Work required: W_by_m = 452.534 **kJ/kg .... Ans.**

COP: COP = 0.068 **...Ans.**

COP_ideal: COP_i = 0.41 **... Carnot effcy .... Ans.**

FOM: FOM = 0.166 **...Ans.**

Download free eBooks at bookboon.com

Plot COP against compressor effcy, as the compressor effcy varies from 0.6 to 1, for eta_HX values of 0.96, 0.98 and 1:

First, write COP as function of eta_co and eta_HX:

$$COP(eta\_co, eta\_HX) := JT\_Refrign\_system\_Ar(P1, T1, P2, eta\_co, eta\_HX)_{1,6}$$

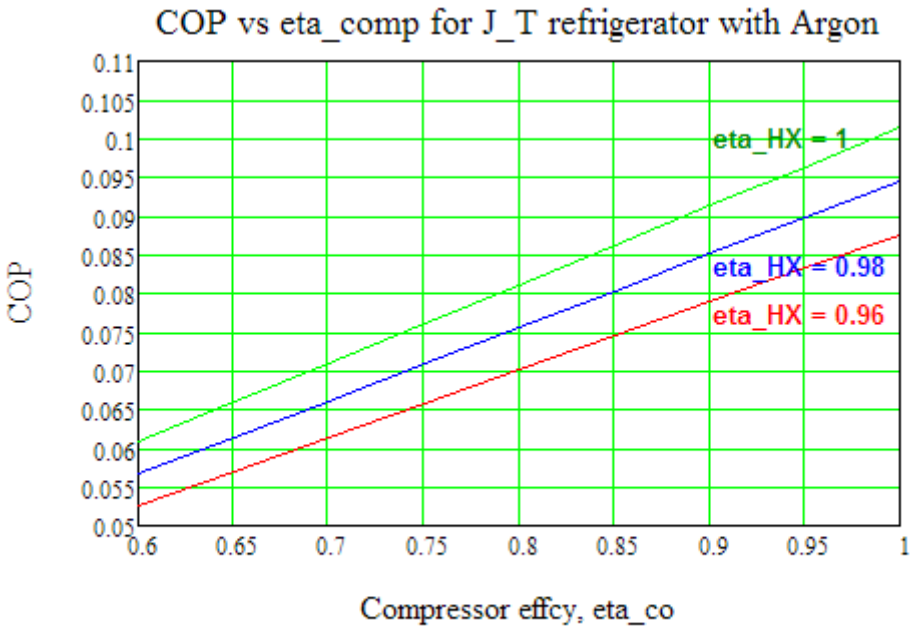
Now:

eta_co := 0.6, 0.65 .. 1 ...define a range variable

And, we get:

eta_co =	COP(eta_co, 0.96)	COP(eta_co, 0.98)	COP(eta_co, 1)
0.6	0.0526	0.0567	0.0608
0.65	0.057	0.0614	0.0659
0.7	0.0613	0.0662	0.071
0.75	0.0657	0.0709	0.0761
0.8	0.0701	0.0756	0.0811
0.85	0.0745	0.0803	0.0862
0.9	0.0789	0.0851	0.0913
0.95	0.0832	0.0898	0.0963
1	0.0876	0.0945	0.1014

And, plot:



==========

Download free eBooks at bookboon.com

Prob. 4.2.15 In Prob.4.2.13, if the working fluid is Nitrogen:

In a J-T Refrigeration system with Nitrogen as working fluid, the system works between 1.013 bar, 300 K and 202.6 bar, with isothermal compression. Efficiencies of compressor and heat exchangers are 72% and 98% respectively. Find out the refrig. effect, work required, COP and FOM.

Compare these quantities with those obtained for Argon in Prob. 4.2.14

Data:

P1 := 1.013 bar T1 := 300 K P2 := 202.6 bar TSAT(P1) = 77.166 K

eta_co := 0.72 ....overall effcy of compressor

eta_HX := 0.98 ..effetiveness of HX

Using the Function written for N2:

JT_Refrign_system_N2(P1, T1, P2, eta_co, eta_HX) =

"P1(bar)"	"T1 (K)"	"Tsat_N2 at P1 (K)"	"P2 (bar)"	"Qa_by_m(kJ/kg)"	"W_by_m(kJ/kg)"	"COP"	"COP_i"	"FOM"
1.013	300	77.166	202.6	27.653	657.545	0.042	0.346	0.121

Download free eBooks at bookboon.com
Click on the ad to read more

Thus:

Refrig. effect: Qa_by_m = 27.653 **kJ/kg .... Ans.**

Work required: W_by_m = 657.545 **kJ/kg .... Ans.**

COP: COP = 0.042 **...Ans.**

COP_ideal: COP_i = 0.346 **... Carnot effcy .... Ans.**

FOM: FOM = 0.121 **...Ans.**

Plot COP against compressor effcy, as the compressor effcy varies from 0.6 to 1, for eta_HX values of 0.96, 0.98 and 1:

First, write COP as function of eta_co and eta_HX:

$$COP(eta\_co, eta\_HX) := JT\_Refrign\_system\_N2(P1, T1, P2, eta\_co, eta\_HX)_{1,6}$$

And:

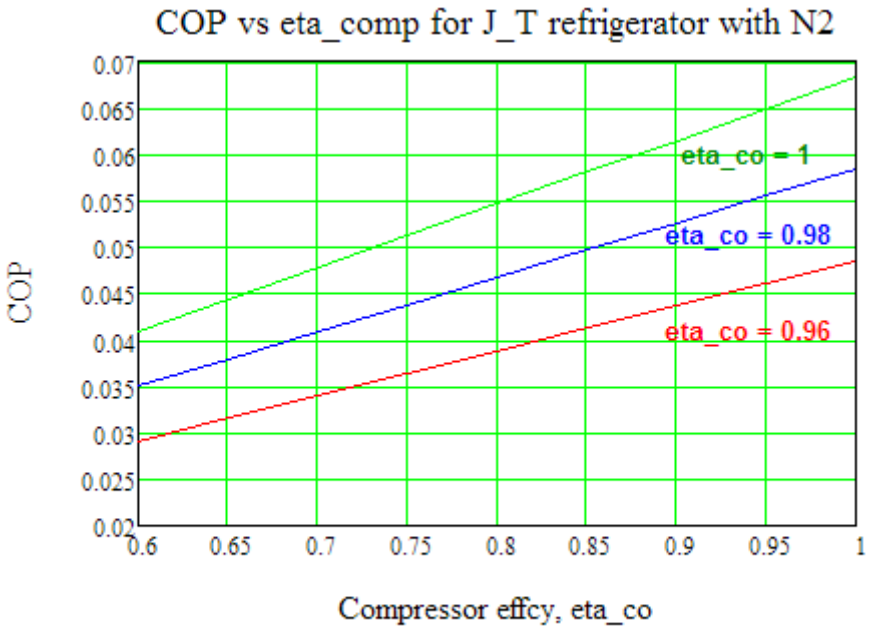
eta_co := 0.6, 0.65 .. 1 ...define a range variable

We get:

eta_co =	COP(eta_co, 0.96)	COP(eta_co, 0.98)	COP(eta_co, 1)
0.6	0.0291	0.035	0.041
0.65	0.0315	0.038	0.0444
0.7	0.034	0.0409	0.0478
0.75	0.0364	0.0438	0.0512
0.8	0.0388	0.0467	0.0546
0.85	0.0412	0.0496	0.0581
0.9	0.0437	0.0526	0.0615
0.95	0.0461	0.0555	0.0649
1	0.0485	0.0584	0.0683

Download free eBooks at bookboon.com

And, plot:

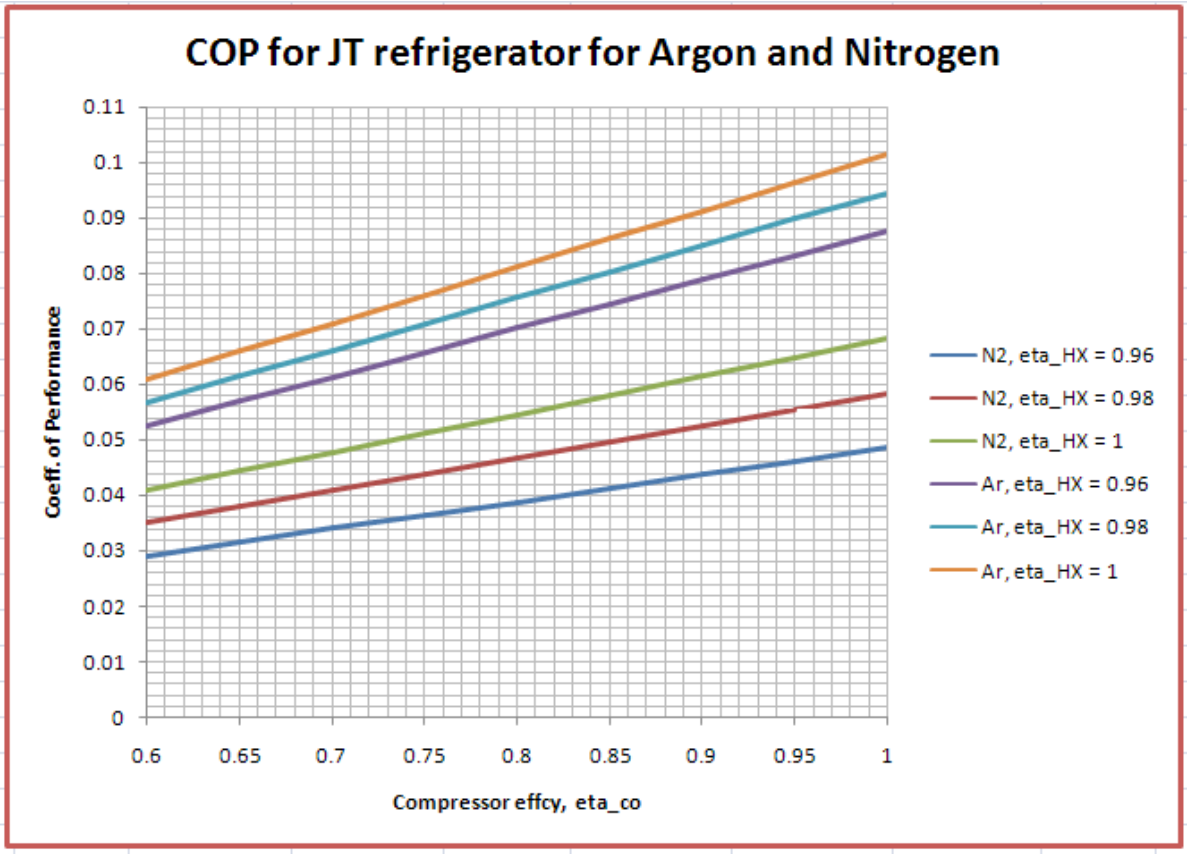


To compare these quantities with those obtained for Argon in Prob.4.2.14:

Transfer the results to EXCEL and plot:

	Coeff of Performance for N2			Coeff of Performance for Argon		
eta_co	eta_HX = 0.96	eta_HX = 0.98	eta_HX = 1	eta_HX = 0.96	eta_HX = 0.98	eta_HX = 1
0.6	0.0291	0.035	0.041	0.0526	0.0567	0.0608
0.65	0.0315	0.038	0.0444	0.057	0.0614	0.0659
0.7	0.034	0.0409	0.0478	0.0613	0.0662	0.071
0.75	0.0364	0.0438	0.0512	0.0657	0.0709	0.0761
0.8	0.0388	0.0467	0.0546	0.0701	0.0756	0.0811
0.85	0.0412	0.0496	0.0581	0.0745	0.0803	0.0862
0.9	0.0437	0.0526	0.0615	0.0789	0.0851	0.0913
0.95	0.0461	0.0555	0.0649	0.0832	0.0898	0.0963
1	0.0485	0.0584	0.0683	0.0876	0.0945	0.1014

Download free eBooks at bookboon.com



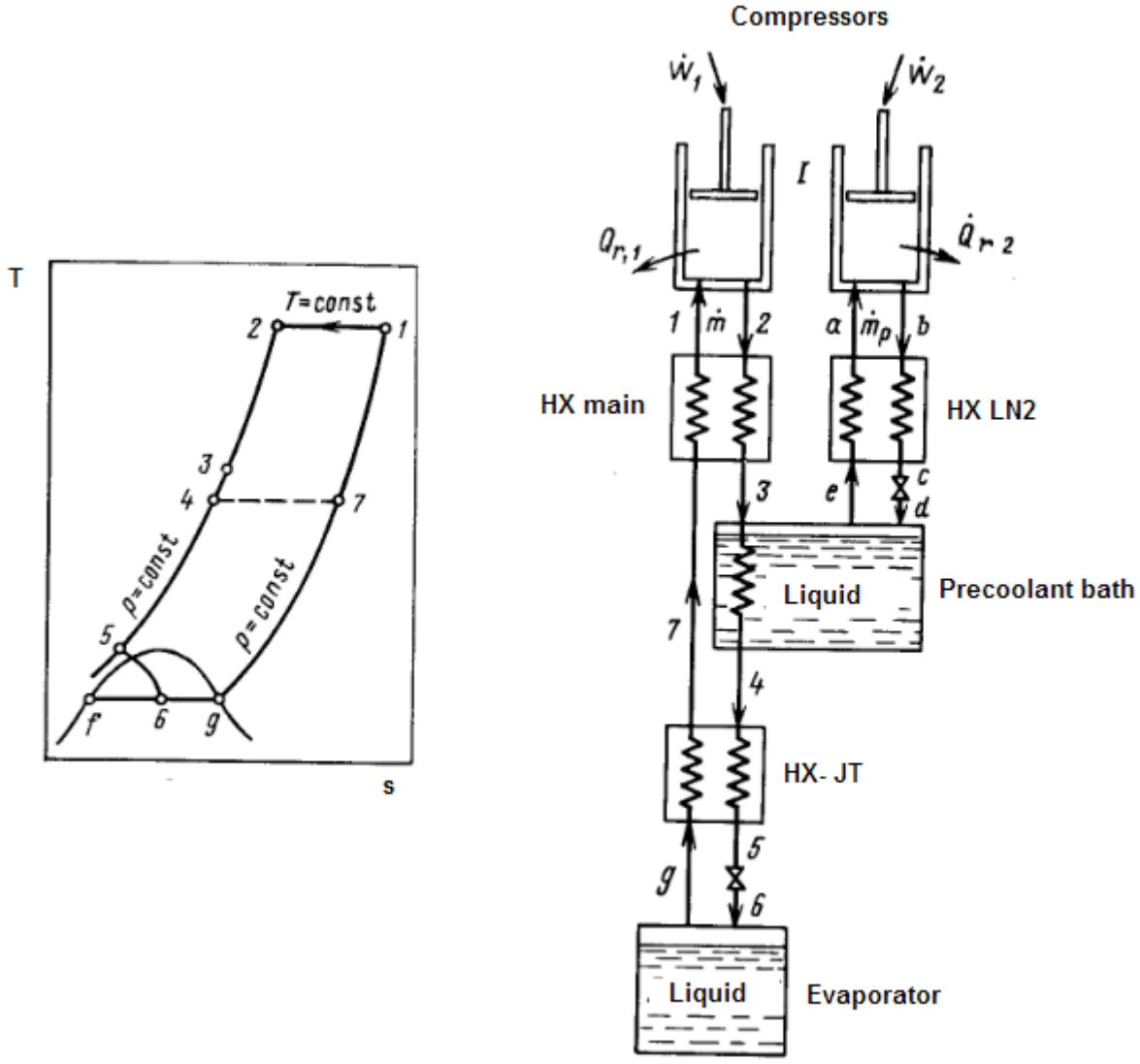
LIGS University
based in Hawaii, USA
is currently enrolling in the Interactive Online BBA, MBA, MSc, DBA and PhD programs:
- enroll by October 31st, 2014 and
- save up to 11% on the tuition!
- pay in 10 installments / 2 years
- Interactive Online education
- visit www.ligsuniversity.com to find out more!

Note: LIGS University is not accredited by any nationally recognized accrediting agency listed by the US Secretary of Education.
More info here.

Download free eBooks at bookboon.com
Click on the ad to read more

Note: COP values for Argon are generally higher than those for Nitrogen, for identical conditions. So, Argon is the preferred to Nitrogen as working fluid for Joule-Thomson refrigerators.

Prob. 4.2.16 Write a Mathcad program to find the various parameters in a pre-cooled JT refrigeration system with H2 as the working fluid and N2 as the pre-coolant.



Note: Temps. in following Function must be in Kelvin, Pressures in bar.

Precooled_JT_H2(P1, T1, P2, P1N2, T1N2, P2N2, eta_cH2, eta_cN2, eta_HXmain, eta_HXN2, eta_HX_JT) :=

Download free eBooks at bookboon.com

$h1 \leftarrow enthalpy\_H2(P1, T1)$

$s1 \leftarrow entropy\_H2(P1, T1)$

$h2 \leftarrow enthalpy\_H2(P2, T1)$

$s2 \leftarrow entropy\_H2(P2, T1)$

$hg \leftarrow HGSATP\_H2(P1)$

$ha \leftarrow enthalpy\_N2(P1N2, T1N2)$

$hb \leftarrow enthalpy\_N2(P2N2, T1N2)$

$sa \leftarrow entropy\_N2(P1N2, T1N2)$

$sb \leftarrow entropy\_N2(P2N2, T1N2)$

$he \leftarrow HGSATP\_N2(P1N2)$

$T4 \leftarrow TSAT\_N2(P1N2)$

$T7 \leftarrow T4$

$h7 \leftarrow enthalpy\_H2(P1, T7)$

$h4 \leftarrow enthalpy\_H2(P2, T7)$

$Qa\_by\_m \leftarrow (h7 - h4) - (1 - eta\_HX\_JT) \cdot (h7 - hg)$

$$z \leftarrow \frac{Qa\_by\_m - [(h1 - h2) - (1 - eta\_HXmain) \cdot (h1 - hg)]}{[(ha - hb) - (1 - eta\_HXN2) \cdot (ha - he)]}$$

$$W\_by\_m \leftarrow \frac{T1 \cdot (s1 - s2) - (h1 - h2)}{eta\_cH2} + z \cdot \frac{[T1N2 \cdot (sa - sb) - (ha - hb)]}{eta\_cN2}$$

$$COP \leftarrow \frac{Qa\_by\_m}{W\_by\_m}$$

$TL \leftarrow TSAT\_H2(P1)$

$$COP_i \leftarrow \frac{TL}{T1 - TL}$$

$$FOM \leftarrow \frac{COP}{COP_i}$$

"P1(bar)"	"T1 (K)"	"Tsat_N2 at P1N2 (K)"	"P2 (bar)"	"z = (m_N2/m_H2)"	"Qa_by_m(kJ/kg)"	"W_by_m(kJ/kg)"	"COP"	"COP_i"	"FOM"
P1	T1	T4	P2	z	Qa_by_m	W_by_m	COP	COP_i	FOM

Prob. 4.2.17 Determine the refrig. effect, COP, and FOM for a pre-cooled JT liquid hydrogen refrigerator, if the hydrogen is compressed between 101.3 kPa and 8.1 MPa and 300 K. The nitrogen is compressed between 101.3 kPa and 15.2 MPa and 300 K. The main HX has an effectiveness of 0.99 and the pre-coolant HX has an effectiveness of 0.98. The cold HX (or the JT HX) in the hydrogen circuit has an effectiveness of 0.96 and both the compressors may be assumed 100% efficient. [1]

Download free eBooks at bookboon.com

Data:

P1 := 1.013 bar T1 := 300 K P2 := 81 bar TSAT_H2(P1) = 20.314 K

P1N2 := 1.013 bar P2N2 := 152 bar T1N2 := 300 K

eta_cH2 := 1 eta_cN2 := 1

eta_HXmain := 0.99 eta_HXN2 := 0.98 eta_HX_JT := 0.96

Using the Mathcad Function written above, we get:

Precooled_JT_H2(P1, T1, P2, P1N2, T1N2, P2N2, eta_cH2, eta_cN2, eta_HXmain, eta_HXN2, eta_HX_JT) =

	0	1	2	3	4
0	"P1(bar)"	"T1 (K)"	"Tsat_N2 at P1N2 (K)"	"P2 (bar)"	"z = (m_N2/m_H2)"
1	1.013	300	77.166	81	9.2654

5	6	7		8	9
"Qa_by_m(kJ/kg)"	"W_by_m(kJ/kg)"		"COP"	"COP_i"	"FOM"
132.635	$9.6258 \cdot 10^3$		0.0138	0.0726	0.1897

TURN TO THE EXPERTS FOR
SUBSCRIPTION CONSULTANCY

Subscrybe is one of the leading companies in Europe when it comes to innovation and business development within subscription businesses.

We innovate new subscription business models or improve existing ones. We do business reviews of existing subscription businesses and we develope acquisition and retention strategies.

Learn more at linkedin.com/company/subscrybe or contact
Managing Director Morten Suhr Hansen at mha@subscrybe.dk

SUBSCRYBE - to the future

Download free eBooks at bookboon.com
Click on the ad to read more

Thus:

Refrig. effect = Qa_by_m = 132.635 kJ/kg … ns.

COP = 0.0138 … Ans.

FOM = 0.1897 … Ans.

Prob. 4.2.18 Determine the refrig. effect, COP, and FOM for a pre-cooled JT liquid hydrogen refrigerator, if the hydrogen is compressed between 1.013 bar and 152 bar and 294 K. The nitrogen is compressed between 1.013 bar and 202.6 bar and 294 K. The nitrogen flow rate ratio (z) is 5. The main HX and the pre-coolant HX have an effectiveness of 100% each. Find the effectiveness of cold HX (or the JT HX) in the hydrogen circuit if both the compressors are 100% efficient. [1]

Solution:

First, assume a value for the effectiveness of HX and apply the Function; obviously the resulting z will not be equal to z = 5. Then, apply the 'Solve block' of Mathcad to get the value of effectiveness of JT HX which will make z = 5:

Data:

P1 := 1.013 bar T1 := 294 K P2 := 152 bar TSAT_H2(P1) = 20.314 K

P1N2 := 1.013 bar P2N2 := 202.6 bar T1N2 := 294 K

eta_cH2 := 1 eta_cN2 := 1 ....compressor efficiencies

eta_HXmain := 1 eta_HXN2 := 1

eta_HX_JT := 1 ....trial value of JT HX

Applying the Mathcad Function:

Precooled_JT_H2(P1, T1, P2, P1N2, T1N2, P2N2, eta_cH2, eta_cN2, eta_HXmain, eta_HXN2, eta_HX_JT) =

	0	1	2	3	4
0	"P1(bar)"	"T1 (K)"	"Tsat_N2 at P1N2 (K)"	"P2 (bar)"	"z = (m_N2/m_H2)"
1	1.013	294	77.166	152	7.832

5	6	7	8	9
"Qa_by_m(kJ/kg)"	"W_by_m(kJ/kg)"	"COP"	"COP_i"	"FOM"
192.788	$9.822 \cdot 10^3$	0.02	0.074	0.264

Download free eBooks at bookboon.com

Note that now z = 7.832.

Now, apply the 'Solve block' of Mathcad to get the value of eta_HX_JT which will make the value of z equal to 5:

z := 5 ...desired value of z

Given

$$Precooled\_JT\_H2(P1, T1, P2, P1N2, T1N2, P2N2, eta\_cH2, eta\_cN2, eta\_HXmain, eta\_HXN2, eta\_HX\_JT)_{1,4} = z$$

eta_HX_JT := Find(eta_HX_JT)

i.e. eta_HX_JT = 0.842 **...value of eta_HX_JT to make z equal to 5 ... Ans.**

Now, to determine other parameters, apply the Function again with the correct value of eta_HX_JT:

Precooled_JT_H2(P1, T1, P2, P1N2, T1N2, P2N2, eta_cH2, eta_cN2, eta_HXmain, eta_HXN2, eta_HX_JT) =

	0	1	2	3	4
0	"P1(bar)"	"T1 (K)"	"Tsat_N2 at P1N2 (K)"	"P2 (bar)"	"z = (m_N2/m_H2)"
1	1.013	294	77.166	152	5

5	6	7	8	9
"Qa_by_m(kJ/kg)"	"W_by_m(kJ/kg)"	"COP"	"COP_i"	"FOM"
96.679	$8.51 \cdot 10^3$	0.011	0.074	0.153

Thus:

Refrig. effect = Qa_by_m = 96.679 kJ/kg … ns.

COP = 0.011 … Ans.

FOM = 0.153 … Ans.

Download free eBooks at bookboon.com

Prob. 4.2.19 Determine the refrig. effect, COP, and FOM for a pre-cooled JT liquid hydrogen refrigerator, if the hydrogen is compressed between 1.013 bar and 101.3 bar and 300 K. The nitrogen is compressed between 1.013 bar and 203 bar and 300 K. The main HX has an effectiveness of 0.98 and the pre-coolant HX has an effectiveness of 0.97. The cold HX (or the JT HX) in the hydrogen circuit has an effectiveness of 0.95 and both the compressors may be assumed to have an efficiency of 75% each. [1]

Data:

P1 := 1.013 bar T1 := 300 K P2 := 101.3 bar TSAT_H2(P1) = 20.314 K

P1N2 := 1.013 bar P2N2 := 203 bar T1N2 := 300 K TSAT_N2(P1N2) = 77.166 K

eta_cH2 := 0.75 eta_cN2 := 0.75 ....compressor efficiencies

eta_HXmain := 0.98 eta_HXN2 := 0.97 eta_HX_JT := 0.95 ... effectivenesses of HX's

"I studied English for 16 years but... ...I finally learned to speak it in just six lessons"
Jane, Chinese architect
ENGLISH OUT THERE
Click to hear me talking before and after my unique course download

Download free eBooks at bookboon.com
Click on the ad to read more

Applying the Mathcad Function:

Precooled_JT_H2(P1, T1, P2, P1N2, T1N2, P2N2, eta_cH2, eta_cN2, eta_HXmain, eta_HXN2, eta_HX_JT) =

	0	1	2	3	4
0	"P1(bar)"	"T1 (K)"	"Tsat_N2 at P1N2 (K)"	"P2 (bar)"	"z = (m_N2/m_H2)"
1	1.013	300	77.166	101.3	10.3909

5	6	7	8	9
"Qa_by_m(kJ/kg)"	"W_by_m(kJ/kg)"	"COP"	"COP_i"	"FOM"
145.5807	$1.4268 \cdot 10^4$	0.0102	0.0726	0.1405

Thus:

Refrig. effect = Qa_by_m = 145.58 kJ/kg … ns.

COP = 0.0102 … Ans.

FOM = 0.1405 … Ans.

Plot the effect of variation of eta_HX_JT on various parameters, if other conditions remain the same:

First, write different parameters as functions of eta_HX_JT:

$z(eta\_HX\_JT) := Precooled\_JT\_H2(P1, T1, P2, P1N2, T1N2, P2N2, eta\_cH2, eta\_cN2, eta\_HXmain, eta\_HXN2, eta\_HX\_JT)_{1,4}$

$Qabym(eta\_HX\_JT) := Precooled\_JT\_H2(P1, T1, P2, P1N2, T1N2, P2N2, eta\_cH2, eta\_cN2, eta\_HXmain, eta\_HXN2, eta\_HX\_JT)_{1,5}$

$Wbym(eta\_HX\_JT) := Precooled\_JT\_H2(P1, T1, P2, P1N2, T1N2, P2N2, eta\_cH2, eta\_cN2, eta\_HXmain, eta\_HXN2, eta\_HX\_JT)_{1,6}$

$COP(eta\_HX\_JT) := Precooled\_JT\_H2(P1, T1, P2, P1N2, T1N2, P2N2, eta\_cH2, eta\_cN2, eta\_HXmain, eta\_HXN2, eta\_HX\_JT)_{1,7}$

$FOM(eta\_HX\_JT) := Precooled\_JT\_H2(P1, T1, P2, P1N2, T1N2, P2N2, eta\_cH2, eta\_cN2, eta\_HXmain, eta\_HXN2, eta\_HX\_JT)_{1,9}$

Now:

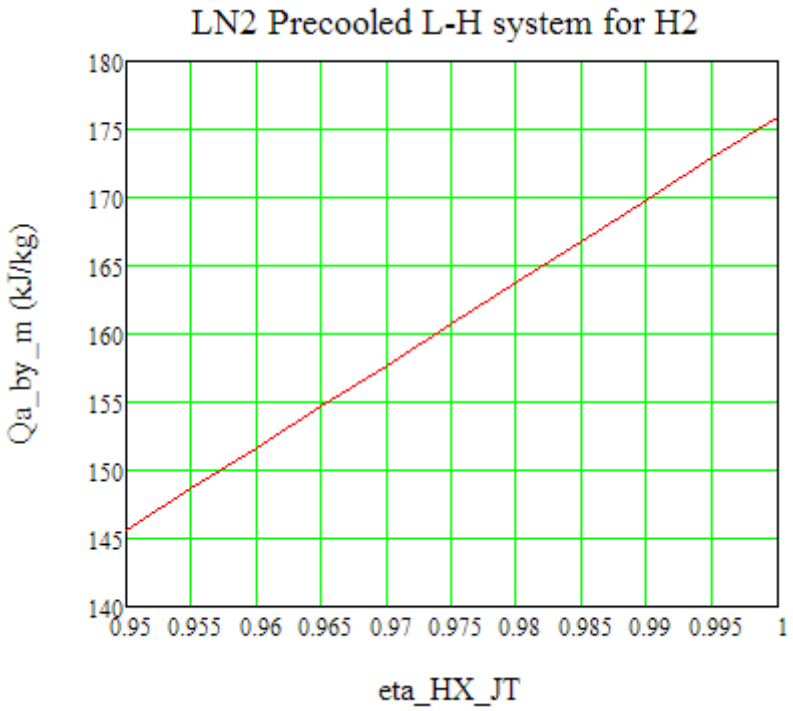
$eta\_HX\_JT := 0.95, 0.955 .. 1$ …define a range variable

Download free eBooks at bookboon.com

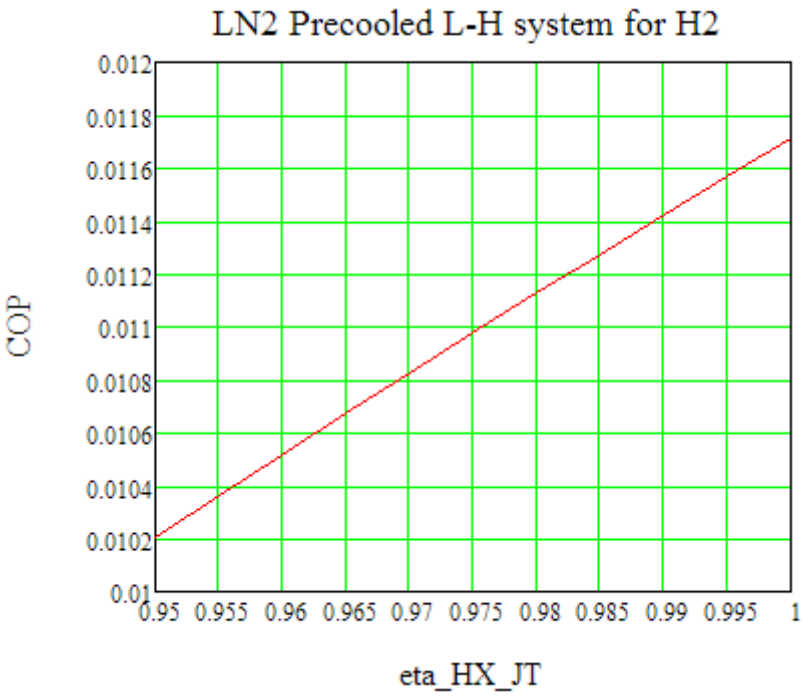
And, we get:

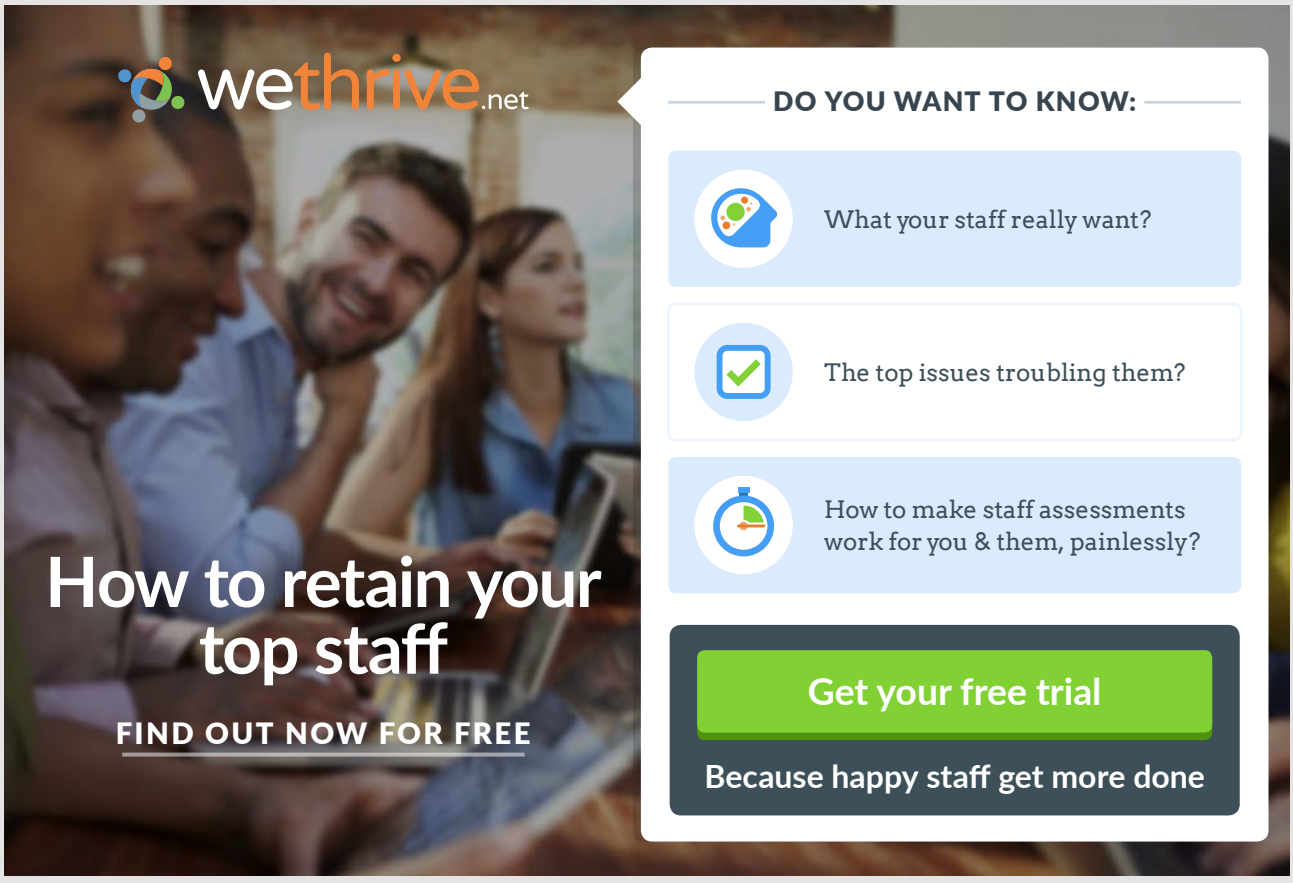
eta_HX_JT =	Qabym(eta_HX_JT)	Wbym(eta_HX_JT)	COP(eta_HX_JT)	FOM(eta_HX_JT)	z(eta_HX_JT)
0.95	145.581	$1.427 \cdot 10^4$	0.0102	0.1405	10.391
0.955	148.615	$1.434 \cdot 10^4$	0.0104	0.1427	10.511
0.96	151.649	$1.442 \cdot 10^4$	0.0105	0.1448	10.63
0.965	154.683	$1.449 \cdot 10^4$	0.0107	0.1469	10.75
0.97	157.717	$1.457 \cdot 10^4$	0.0108	0.149	10.87
0.975	160.751	$1.465 \cdot 10^4$	0.011	0.1511	10.989
0.98	163.785	$1.472 \cdot 10^4$	0.0111	0.1532	11.109
0.985	166.819	$1.48 \cdot 10^4$	0.0113	0.1552	11.229
0.99	169.853	$1.487 \cdot 10^4$	0.0114	0.1572	11.349
0.995	172.887	$1.495 \cdot 10^4$	0.0116	0.1592	11.468
1	175.922	$1.502 \cdot 10^4$	0.0117	0.1612	11.588

Now, plot the results:

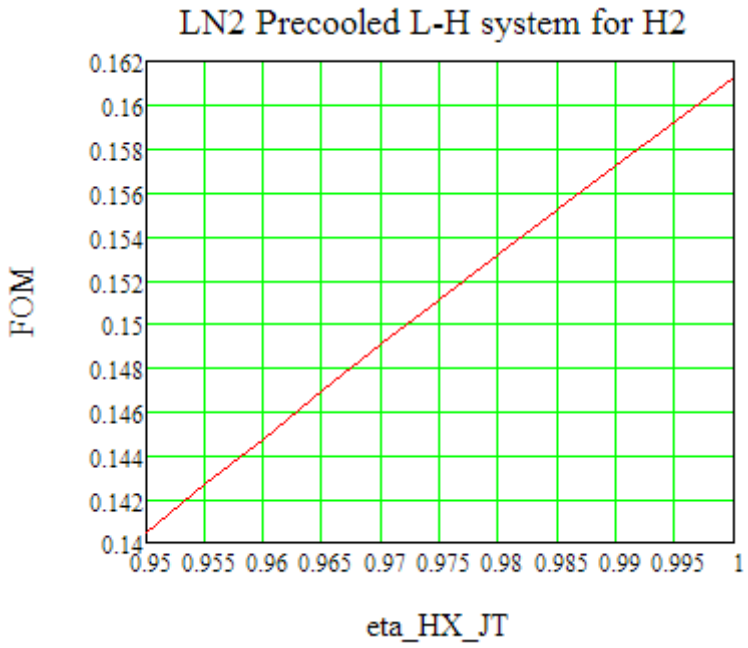


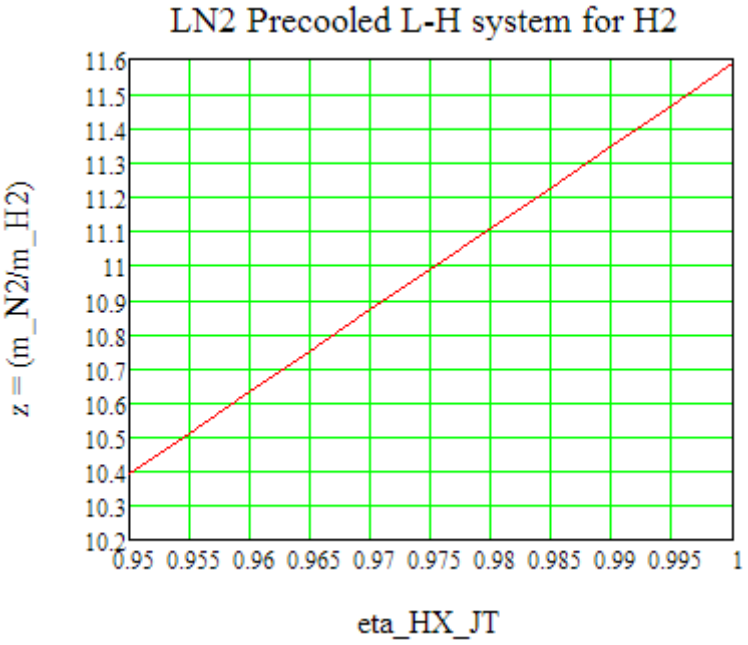
Download free eBooks at bookboon.com




wethrive.net
How to retain your top staff
FIND OUT NOW FOR FREE
DO YOU WANT TO KNOW:
What your staff really want?
The top issues troubling them?
How to make staff assessments work for you & them, painlessly?
Get your free trial
Because happy staff get more done

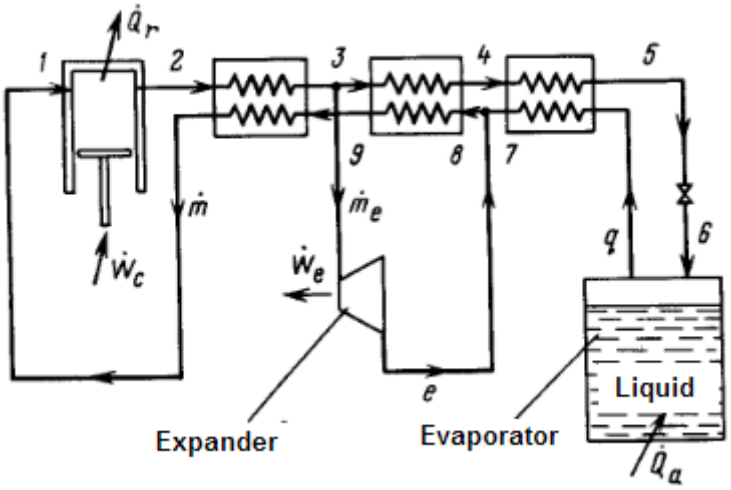
Download free eBooks at bookboon.com
Click on the ad to read more

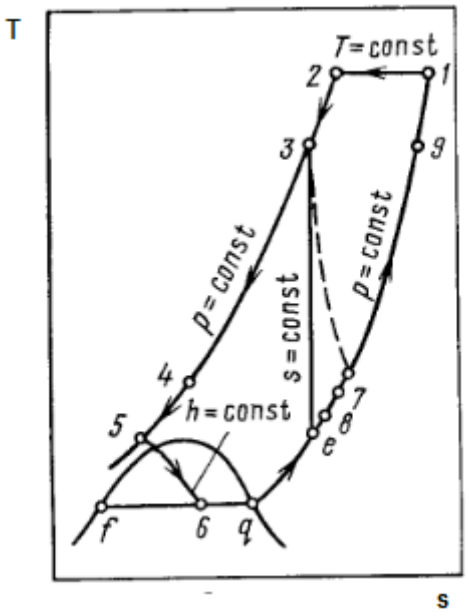




Prob. 4.2.20 Write a Mathcad program to find the various parameters in a Claude refrigeration system, where a fraction x of the gas compressed in the compressor is sent through the expander.

Download free eBooks at bookboon.com





Claude_refrig_N2(P1, T1, P2, T3, eta_co, eta_e_ad, eta_em, Fraction_x) :=

h1 ← enthalpy_N2(P1, T1)
s1 ← entropy_N2(P1, T1)
h2 ← enthalpy_N2(P2, T1)
s2 ← entropy_N2(P2, T1)
h3 ← enthalpy_N2(P2, T3)
s3 ← entropy_N2(P2, T3)
se ← s3

Download free eBooks at bookboon.com

$$he \leftarrow enthalpy\_N2\_Ps(P1, se)$$

$$Qa\_by\_m \leftarrow (h1 - h2) + Fraction\_x \cdot eta\_e\_ad \cdot (h3 - he)$$

$$W\_by\_m \leftarrow \frac{T1 \cdot (s1 - s2) - (h1 - h2)}{eta\_co} - Fraction\_x \cdot eta\_e\_ad \cdot eta\_em \cdot (h3 - he)$$

$$COP \leftarrow \frac{Qa\_by\_m}{W\_by\_m}$$

$$TL \leftarrow TSAT\_N2(P1)$$

$$COP_i \leftarrow \frac{TL}{T1 - TL}$$

$$FOM \leftarrow \frac{COP}{COP_i}$$

"P1(bar)"	"T1 (K)"	"Tsat_N2 at P1 (K)"	"P2 (bar)"	"x = (m_exp/m_comp)"	"Qa_by_m(kJ/kg)"	"W_by_m(kJ/kg)"	"COP"	"COP_i"	"FOM"
P1	T1	TL	P2	Fraction_x	Qa_by_m	W_by_m	COP	COP_i	FOM

Prob. 4.2.21 In an ideal Claude refrigeration system using nitrogen as the working fluid, the gas enters the reversible isothermal compressor at 1.013 bar, and 300 K, and is compressed to 40.5 bar. Determine the expander mass flow rate ratio (x = me/m) required to result in a refrigeration effect of 80 kJ/kg, if the gas enters the reversible adiabatic expander at 4.5 bar and 240 K. [1]

Download free eBooks at bookboon.com
Click on the ad to read more

Solution:

First, assume a trial value for Fraction_x and apply the Function. Obviously, the refrig. effect will not be 80 kJ/kg. Then, use the 'Solve block' of Mathcad to get the correct value of Fraction_x that will make the refrig. effect 80 kJ/kg:

Data:

P1 := 1.013 bar T1 := 300 K P2 := 40.5 bar TSAT_N2(P1) = 77.166 K

T3 := 240 K

eta_co := 1 eta_e_ad := 1 eta_em := 1

Fraction_x := 0.1 ....trial value

Applying the Function written above, we get:

Claude_refrig_N2(P1, T1, P2, T3, eta_co, eta_e_ad, eta_em, Fraction_x) =

	0	1	2	3	4
0	"P1(bar)"	"T1 (K)"	"Tsat_N2 at P1 (K)"	"P2 (bar)"	"x = (m_exp/m_comp)"
1	1.013	300	77.166	40.5	0.1

5	6	7	8	9
"Qa_by_m(kJ/kg)"	"W_by_m(kJ/kg)"	"COP"	"COP_i"	"FOM"
23.685	313.085	0.076	0.346	0.218

Thus, we see that with the assumed value of Fraction_x, the refrig. Effect is: Qa_by_m = 23.685 kJ/kg.

Now, use the Solve block of Mathcad:

Qabym := 80 kJ/kg

Given

$\text{Claude\_refrig\_N2}(P1, T1, P2, T3, \text{eta\_co}, \text{eta\_e\_ad}, \text{eta\_em}, \text{Fraction\_x})_{1,5} = \text{Qabym}$

Fraction_x := Find(Fraction_x)

i.e. Fraction_x = 0.467 **...fraction to pass through the expander to obtain a refrig. effect of 80 kJ/kg ... Ans.**

Download free eBooks at bookboon.com

To determine other quantities, with this correct value of Fraction_x, apply the Function again:

Claude_refrig_N2(P1, T1, P2, T3, eta_co, eta_e_ad, eta_em, Fraction_x) =

	0	1	2	3	4
0	"P1(bar)"	"T1 (K)"	"Tsat_N2 at P1 (K)"	"P2 (bar)"	"x = (m_exp/m_comp)"
1	1.013	300	77.166	40.5	0.467

5	6	7	8	9
"Qa_by_m(kJ/kg)"	"W_by_m(kJ/kg)"	"COP"	"COP_i"	"FOM"
80	256.769	0.312	0.346	0.9

Thus:

Qa_by_m = 80 kJ/kg … Ans.

COP = 0.312 … Ans.

FOM = 0.9 …. Ans.

Plot the refrig. effect and COP against Fraction_x as Fraction_x varies from 0.1 to 0.6:

Write the relevant quantities as functions of Fraction_x:

$Q\_by\_m(Fraction\_x) := Claude\_refrig\_N2(P1, T1, P2, T3, eta\_co, eta\_e\_ad, eta\_em, Fraction\_x)_{1,5}$

$COP(Fraction\_x) := Claude\_refrig\_N2(P1, T1, P2, T3, eta\_co, eta\_e\_ad, eta\_em, Fraction\_x)_{1,7}$

Now:

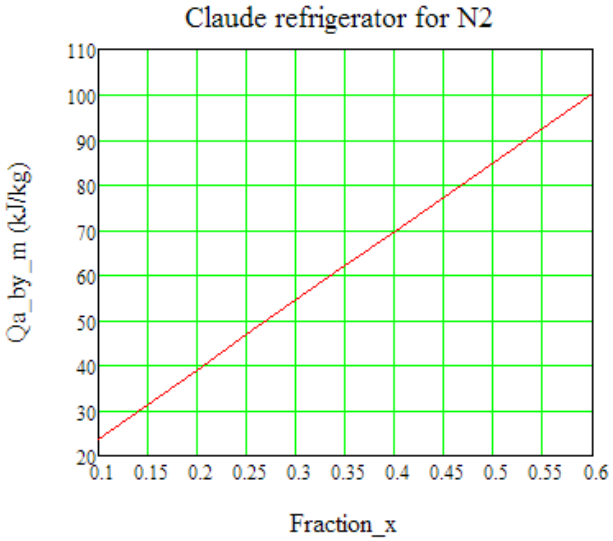
Fraction_x := 0.1, 0.15 .. 0.6 ....define a range variable

And, we get:

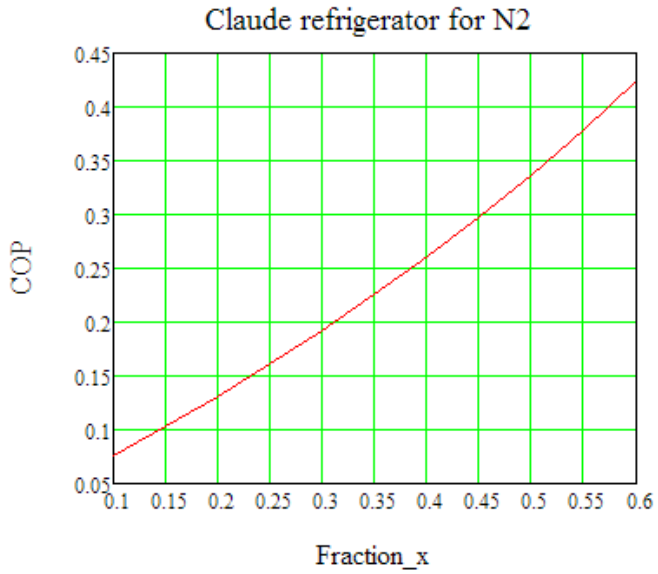
Fraction_x =	Q_by_m(Fraction_x)	COP(Fraction_x)
0.1	23.685	0.076
0.15	31.349	0.103
0.2	39.014	0.131
0.25	46.679	0.161
0.3	54.343	0.192
0.35	62.008	0.226
0.4	69.673	0.261
0.45	77.337	0.298
0.5	85.002	0.338
0.55	92.667	0.38
0.6	100.331	0.424

Download free eBooks at bookboon.com

And, plot:



This e-book
is made with
SetaPDF
SETASIGN
PDF components for PHP developers
www.setasign.com
Download free eBooks at bookboon.com
Click on the ad to read more



==========

Prob.4.22 In the above problem, if the compressor overall efficiency is 75%, adiabatic efficiency and mech. efficiency of expander are 80% and 90%, other quantities remaining the same, find the refrig. effect and COP.

Data:

P1 := 1.013 bar T1 := 300 K P2 := 40.5 bar TSAT_N2(P1) = 77.166 K

T3 := 240 K

eta_co := 0.75 eta_e_ad := 0.8 eta_em := 0.9

Fraction_x := 0.467

Applying the Function for Claude refrigerator:

Claude_refrig_N2(P1, T1, P2, T3, eta_co, eta_e_ad, eta_em, Fraction_x) =

	0	1	2	3	4	
0	"P1(bar)"	"T1 (K)"	"Tsat_N2 at P1 (K)"	"P2 (bar)"	"x = (m_exp/m_comp)"	
1	1.013	300	77.166	40.5		0.467

5	6	7		8	9
"Qa_by_m(kJ/kg)"	"W_by_m(kJ/kg)"	"COP"		"COP_i"	"FOM"
65.6258	386.3422	0.1699		0.3463	0.4905

Download free eBooks at bookboon.com

Thus:

Refrig. effect: Qa_by_m = 65.626 **kJ/kg .... Ans.**

COP: COP = 0.17 **....Ans.**

Plot the refrig. effect and COP against Fraction_x as Fraction_x varies from 0.1 to 0.6:

Write the relevant quantities as functions of Fraction_x:

$$Q\_by\_m(Fraction\_x) := Claude\_refrig\_N2(P1, T1, P2, T3, eta\_co, eta\_e\_ad, eta\_em, Fraction\_x)_{1,5}$$

$$COP(Fraction\_x) := Claude\_refrig\_N2(P1, T1, P2, T3, eta\_co, eta\_e\_ad, eta\_em, Fraction\_x)_{1,7}$$

Now:

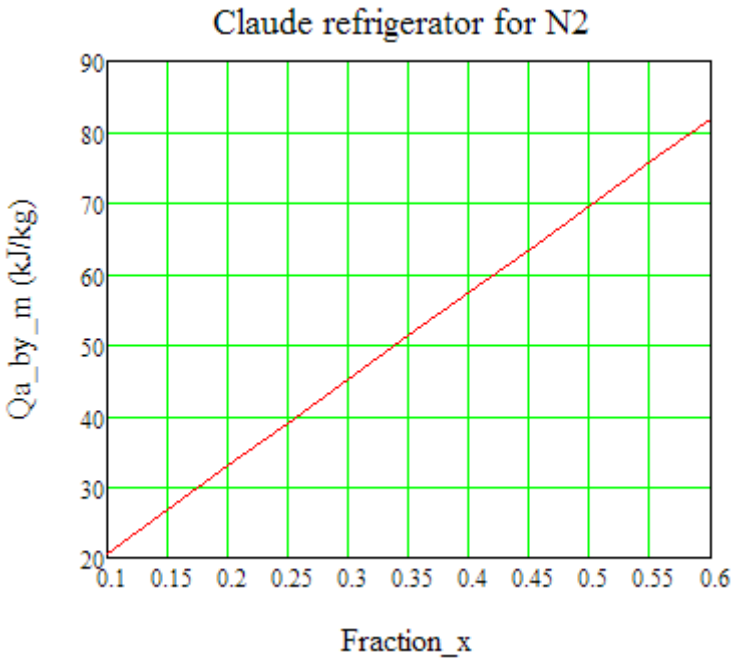
Fraction_x := 0.1, 0.15 .. 0.6 ....define a range variable

And, we get:

Fraction_x =	Q_by_m(Fraction_x)	COP(Fraction_x)
0.1	20.619	0.0483
0.15	26.751	0.0635
0.2	32.882	0.0791
0.25	39.014	0.0951
0.3	45.146	0.1115
0.35	51.277	0.1284
0.4	57.409	0.1458
0.45	63.541	0.1637
0.5	69.673	0.1821
0.55	75.804	0.201
0.6	81.936	0.2205

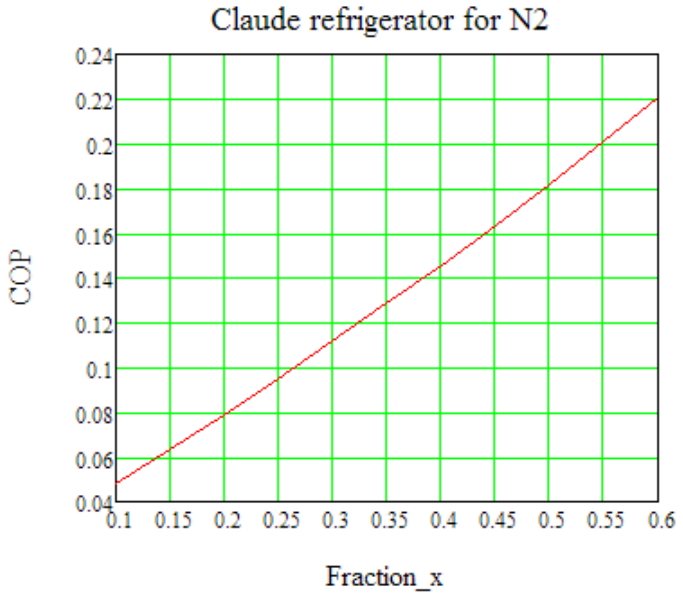
Download free eBooks at bookboon.com

And, plot the results:



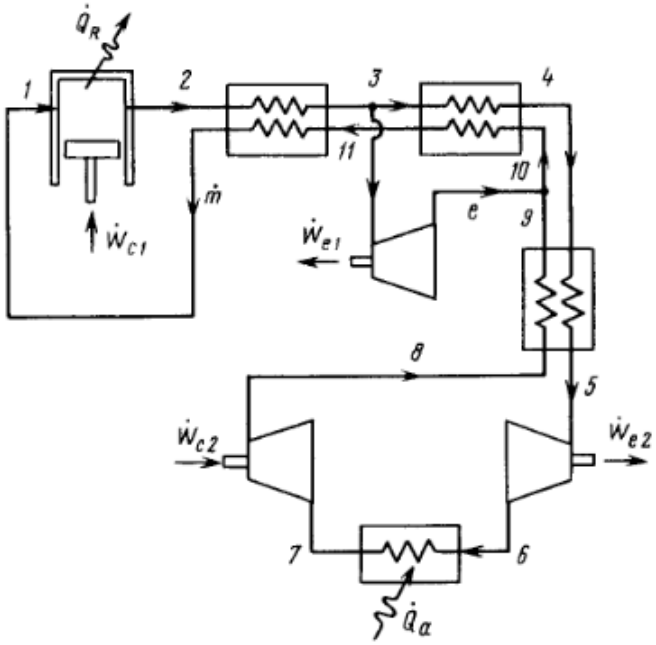
Deloitte.
© Deloitte & Touche LLP and affiliated entities.
Discover the truth at www.deloitte.ca/careers

Download free eBooks at bookboon.com
Click on the ad to read more

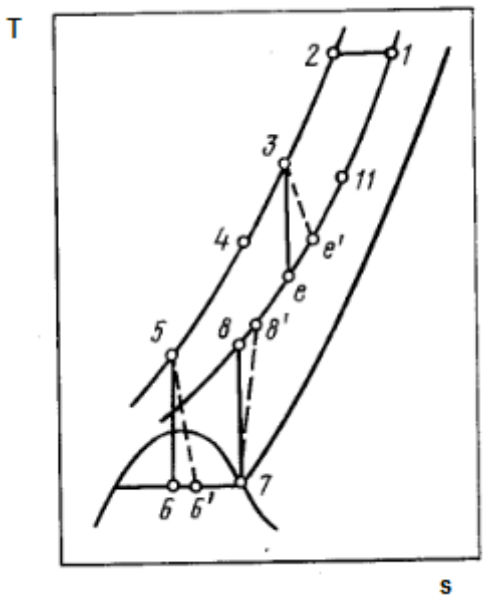


==========

Prob.4.2.23 In a refrigeration system with a wet gas expander (see the fig.), the working fluid is helium, which enters the main compressor at 270 K and 4.053 bar. Compressors and expanders may be assumed to have efficiencies as shown below. At point 3, 25% of the main stream is by-passed to the main expander at a temp of 200 K and 40.53 bar. The helium gas leaves the main compressor at 300 K and 40.53 bar. The helium enters the low temp compressor at 1.013 bar and sat. vapor conditions and leaves at 4.053 bar. Determine the coeff. of performance and the FOM. [1]



Download free eBooks at bookboon.com



First, write a Mathcad Function to calculate various parameters for a "Cold gas refrigeration system":

```
Coldgas_Refrig_He(P1,T1,P2,T2,T3,P7,Fraction_x,eta_co,eta_c,eta_ad_e1,eta_m_e1,eta_ad_e2) :=

h1 ← enthalpy_He(P1,T1)
s1 ← entropy_He(P1,T1)
h2 ← enthalpy_He(P2,T2)
s2 ← entropy_He(P2,T2)
P3 ← P2
P5 ← P2
P8 ← P1
T7 ← TSAT_He(P7)
h3 ← enthalpy_He(P3,T3)
s3 ← entropy_He(P3,T3)
se ← s3
he ← enthalpy_He_Ps(P8,se)
h7 ← HGSATP_He(P7)
s7 ← SGSATP_He(P7)
sg ← s7
s8 ← s7
h8 ← enthalpy_He_Ps(P8,s8)
```

Download free eBooks at bookboon.com

$$sf \leftarrow SFSATP\_He(P7)$$

$$he\_prime \leftarrow h3 - eta\_ad\_e1 \cdot (h3 - he)$$

$$h8\_prime \leftarrow h7 + \frac{(h8 - h7)}{eta\_c}$$

$$h5 \leftarrow \frac{(h2 - h1) - Fraction\_x \cdot (h3 - he\_prime)}{1 - Fraction\_x} + h8\_prime$$

$$s5 \leftarrow entropy\_He\_Ph(P5, h5)$$

$$x6 \leftarrow \frac{sg - s5}{sg - sf}$$

$$h6 \leftarrow enthalpy\_2phase\_Px\_He(P7, x6)$$

$$h6\_prime \leftarrow h5 - eta\_ad\_e2 \cdot (h5 - h6)$$

$$Qa\_by\_m \leftarrow (1 - Fraction\_x) \cdot (h7 - h6\_prime)$$

$$Wc1\_by\_m \leftarrow \frac{T2 \cdot (s1 - s2) - (h1 - h2)}{eta\_co}$$

$$Wc2\_by\_m \leftarrow (1 - Fraction\_x) \cdot (h8\_prime - h7)$$

$$We1\_by\_m \leftarrow Fraction\_x \cdot eta\_m\_e1 \cdot (h3 - he\_prime)$$

$$Wnet\_by\_m \leftarrow Wc1\_by\_m + Wc2\_by\_m - We1\_by\_m$$

be > your degree
© 2013 Accenture. All rights reserved.
Bring your talent and passion to a global organization at the forefront of business, technology and innovation. Discover how great you can be.
Visit accenture.com/bookboon
Be greater than.
consulting | technology | outsourcing
accenture
High performance. Delivered.
Download free eBooks at bookboon.com
Click on the ad to read more

$$COP \leftarrow \frac{Qa\_by\_m}{Wnet\_by\_m}$$

$$COP\_i \leftarrow \frac{T7}{T2 - T7}$$

$$FOM \leftarrow \frac{COP}{COP\_i}$$

"T7 (K)"	"We2_by_m"	"Qa_by_m"	"We1_by_m"	"Wc1_by_m"	"Wc2_by_m"	"Wnet_by_m(kJ/kg)"	"COP"	"COP_i"	"FOM"
T7	eta_ad_e2·(h5 − h6)	Qa_by_m	We1_by_m	Wc1_by_m	Wc2_by_m	Wnet_by_m	COP	COP_i	FOM

Now, solve the problem:

Data:

P1 := 4.053 bar T1 := 270 K P2 := 40.53 bar T2 := 300 K

P3 := P2 T3 := 200 K Fraction_x := 0.25 P5 := P2

P7 := 1.013 bar T7 := TSAT_He(P7) i.e. T7 = 4.229 K

P8 := P1

Efficiencies:

eta_co := 0.7 ...effcy of main compressor

eta_ad_e1 := 0.85 ....adiab. effcy of expander 1

eta_m_e1 := 0.95 ....mech. effcy of expander 1

eta_c := 0.8 ..adiab. effcy of low temp compressor

eta_ad_e2 := 0.85 ....adiab. effcy of wet expander

Download free eBooks at bookboon.com

Now, apply the Function written above:

Coldgas_Refrig_He(P1, T1, P2, T2, T3, P7, Fraction_x, eta_co, eta_c, eta_ad_e1, eta_m_e1, eta_ad_e2) =

	0	1	2	3	4
0	"T7 (K)"	"We2_by_m"	"Qa_by_m"	"We1_by_m"	"Wc1_by_m"
1	4.229	67.851	5.493	131.026	$2.122 \cdot 10^{3}$

5	6	7	8	9
"Wc2_by_m"	"Wnet_by_m(kJ/kg)"	"COP"	"COP_i"	"FOM"
15.748	$2.007 \cdot 10^{3}$	$2.737 \cdot 10^{-3}$	0.014	0.191

Thus:

Refrig. Effect = Qa_by_m = 5.493 kJ/kg … Ans.

COP = 0.002737 … Ans.

FOM = 0.191 … Ans.

Plot Refrig. effect, COP and FOM as P2 varies from 20 to 50 bar for expander flow rate ratios of 0.1, 0.2 and 0.25, other factors remaining the same:

First, write relevant quantities as functions of P2 and Qa_by_m:

$$Qa\_by\_m(P2, Fraction\_x) := Coldgas\_Refrig\_He(P1, T1, P2, T2, T3, P7, Fraction\_x, eta\_co, eta\_c, eta\_ad\_e1, eta\_m\_e1, eta\_ad\_e2)_{1,2}$$

$$COP(P2, Fraction\_x) := Coldgas\_Refrig\_He(P1, T1, P2, T2, T3, P7, Fraction\_x, eta\_co, eta\_c, eta\_ad\_e1, eta\_m\_e1, eta\_ad\_e2)_{1,7}$$

$$FOM(P2, Fraction\_x) := Coldgas\_Refrig\_He(P1, T1, P2, T2, T3, P7, Fraction\_x, eta\_co, eta\_c, eta\_ad\_e1, eta\_m\_e1, eta\_ad\_e2)_{1,9}$$

Effect of P2 when Fraction_x remains as 0.25:

P2 := 20, 25 .. 50 …define a range variable

Download free eBooks at bookboon.com

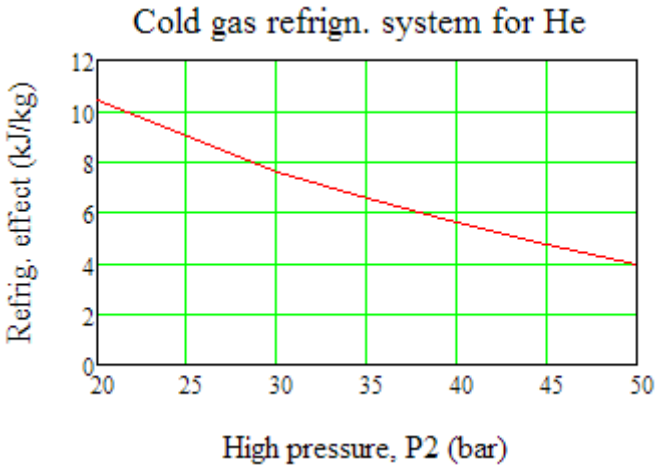
We get:

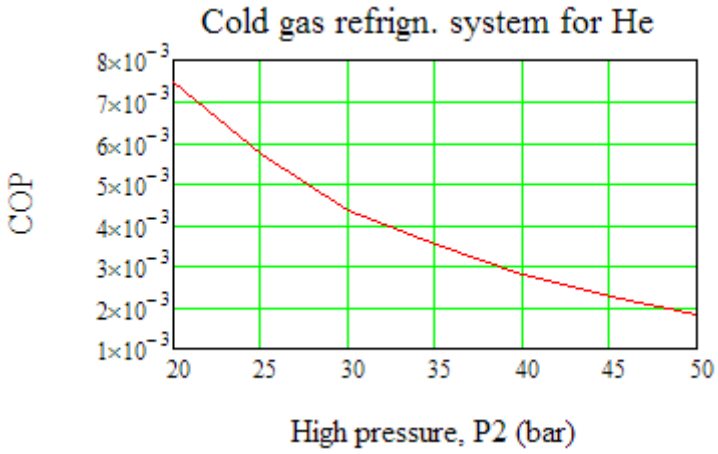
P2 =	Qa_by_m(P2,Fraction_x)	COP(P2,Fraction_x)	FOM(P2,Fraction_x)
20	10.453	$7.473 \cdot 10^{-3}$	0.523
25	9.009	$5.729 \cdot 10^{-3}$	0.401
30	7.594	$4.347 \cdot 10^{-3}$	0.304
35	6.58	$3.516 \cdot 10^{-3}$	0.246
40	5.578	$2.794 \cdot 10^{-3}$	0.195
45	4.776	$2.281 \cdot 10^{-3}$	0.16
50	3.986	$1.819 \cdot 10^{-3}$	0.127

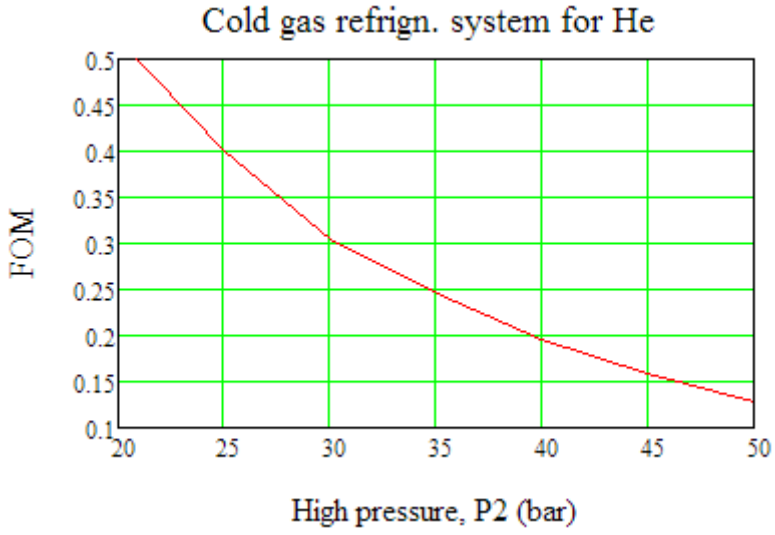
The Wake
the only emission we want to leave behind
Low-speed Engines Medium-speed Engines Turbochargers Propellers Propulsion Packages PrimeServ
The design of eco-friendly marine power and propulsion solutions is crucial for MAN Diesel & Turbo.
Power competencies are offered with the world's largest engine programme – having outputs spanning
from 450 to 87,220 kW per engine. Get up front!
Find out more at www.mandieselturbo.com
Engineering the Future – since 1758.
MAN Diesel & Turbo
MAN

Download free eBooks at bookboon.com
Click on the ad to read more

Plots:





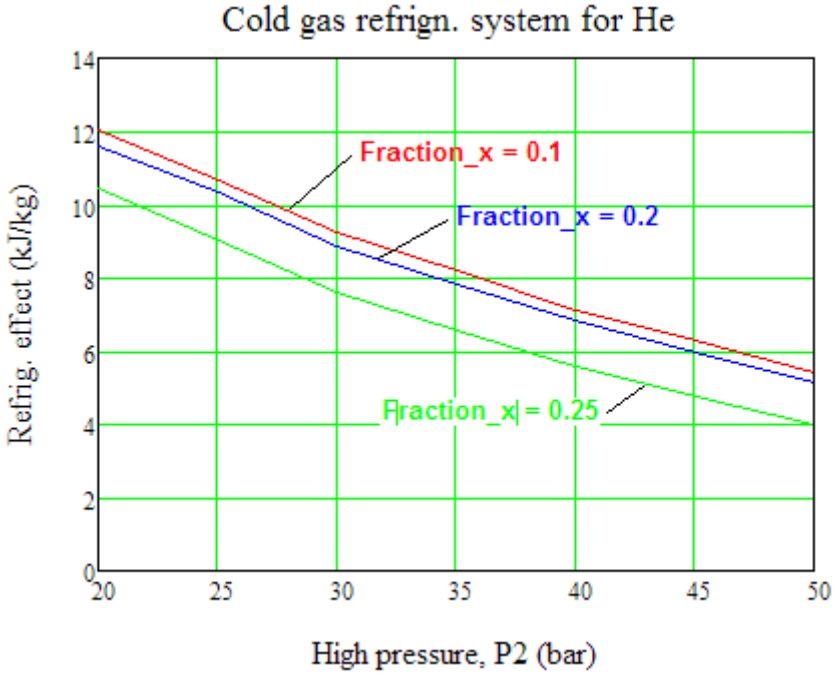


Download free eBooks at bookboon.com

Effect of P2 for Fraction_x = 0.1, 0.2 and 0.25:

Refrig. Effect:

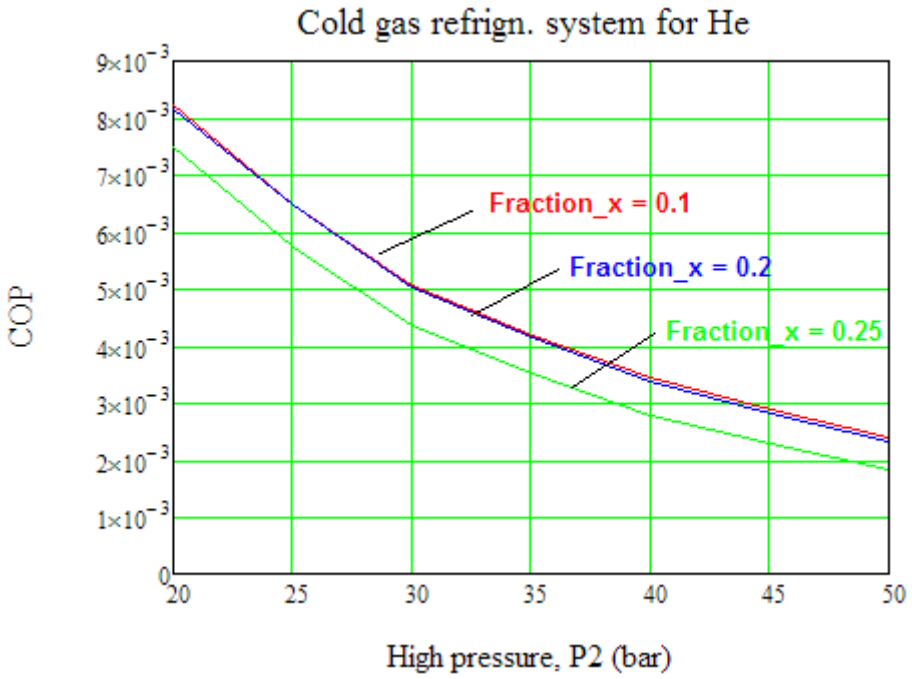
P2 =	Qa_by_m(P2, 0.1)	Qa_by_m(P2, 0.2)	Qa_by_m(P2, 0.25)
20	12.036	11.58	10.453
25	10.637	10.32	9.009
30	9.229	8.88	7.594
35	8.18	7.84	6.58
40	7.128	6.81	5.578
45	6.271	5.981	4.776
50	5.411	5.163	3.986



COP:

P2 =	COP(P2, 0.1) =	COP(P2, 0.2) =	COP(P2, 0.25) =
20	$8.222 \cdot 10^{-3}$	$8.152 \cdot 10^{-3}$	$7.473 \cdot 10^{-3}$
25	$6.475 \cdot 10^{-3}$	$6.467 \cdot 10^{-3}$	$5.729 \cdot 10^{-3}$
30	$5.065 \cdot 10^{-3}$	$5.011 \cdot 10^{-3}$	$4.347 \cdot 10^{-3}$
35	$4.195 \cdot 10^{-3}$	$4.131 \cdot 10^{-3}$	$3.516 \cdot 10^{-3}$
40	$3.43 \cdot 10^{-3}$	$3.365 \cdot 10^{-3}$	$2.794 \cdot 10^{-3}$
45	$2.879 \cdot 10^{-3}$	$2.819 \cdot 10^{-3}$	$2.281 \cdot 10^{-3}$
50	$2.375 \cdot 10^{-3}$	$2.325 \cdot 10^{-3}$	$1.819 \cdot 10^{-3}$

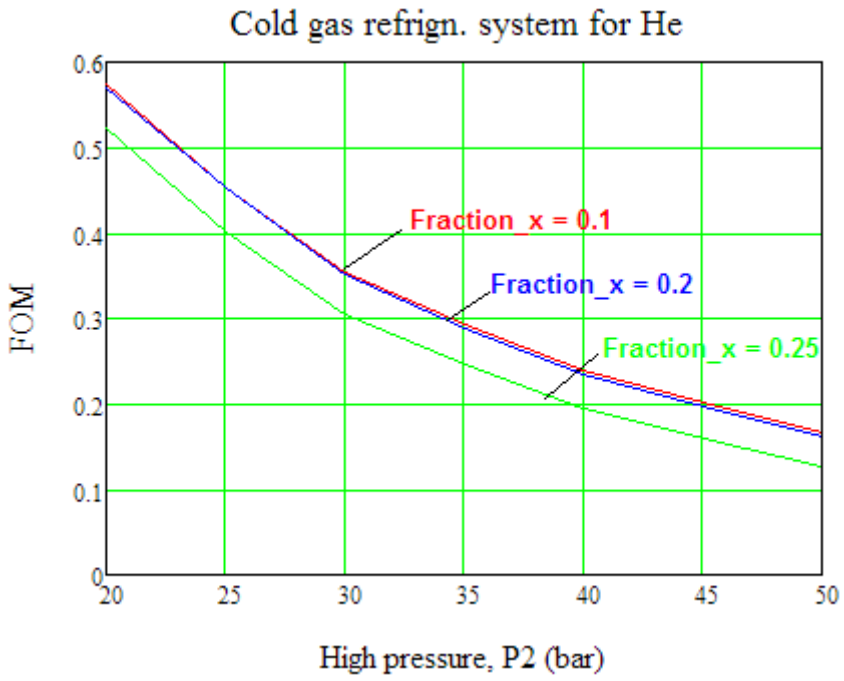
Download free eBooks at bookboon.com



SMS from your computer
...Sync'd with your Android phone & number
FREE 30 days trial!
Go to
BrowserTexting.com
and start texting from your computer!
BrowserTexting
Download free eBooks at bookboon.com
Click on the ad to read more

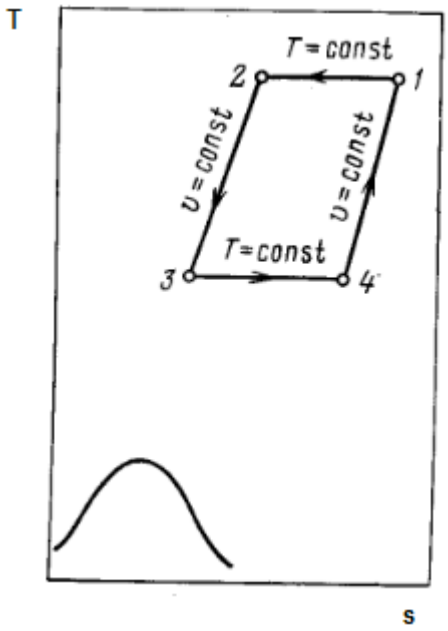
FOM:

P2 =	FOM(P2, 0.1)	FOM(P2, 0.2)	FOM(P2, 0.25)
20	0.575	0.57	0.523
25	0.453	0.452	0.401
30	0.354	0.35	0.304
35	0.293	0.289	0.246
40	0.24	0.235	0.195
45	0.201	0.197	0.16
50	0.166	0.163	0.127



Prob.4.2.24 A Philips refrigerator is used to maintain 100 K while rejecting heat to an ambient sink at 300 K. The working fluid is hydrogen gas which may be considered to behave as an ideal gas. The minimum pressure in the system (point 4) is 1.013 bar and the maximum pressure (point 2) is 10.13 bar. Determine the refrign. effect per unit mass, the COP and the FOM, if all the processes are ideal. [1]

Download free eBooks at bookboon.com



First, write a Mathcad Function to calculate various parameters for Ideal Philips refrigerator working on Stirling cycle:

Ideal_Philips_Refrig_H2(P_min, P_max, TH, TL) :=

P2 ← P_max
P4 ← P_min
T1 ← TH
T2 ← TH
T3 ← TL
T4 ← TL
$P1 \leftarrow T1 \cdot \left(\frac{P4}{T4}\right)$
$P3 \leftarrow T3 \cdot \left(\frac{P2}{T2}\right)$
s1 ← entropy_H2(P1, T1)
s2 ← entropy_H2(P2, T2)
s3 ← entropy_H2(P3, T3)
s4 ← entropy_H2(P4, T4)
Qrej_by_m ← T1·(s1 − s2)
Qa_by_m ← T3·(s4 − s3)

Download free eBooks at bookboon.com

$$W\_net\_by\_m \leftarrow Qrej\_by\_m - Qa\_by\_m$$

$$COP \leftarrow \frac{Qa\_by\_m}{W\_net\_by\_m}$$

$$COP\_i \leftarrow \frac{T3}{T1 - T3}$$

$$FOM \leftarrow \frac{COP}{COP\_i}$$

"P1(bar)"	"P2 (bar)"	"P3 (bar)"	"P4 (bar)"	"Qa_by_m(kJ/kg)"	"W_net_by_m(kJ/kg)"	"COP"	"COP_i"	"FOM"
P1	P2	P3	P4	Qa_by_m	W_net_by_m	COP	COP_i	FOM

Now, solve the problem:

Data:

$P\_min := 1.013$ bar $\quad P\_max := 10.13$ bar $\quad TH := 300$ K

$TL := 100$ K

Download free eBooks at bookboon.com

Click on the ad to read more

Applying the Function written above, we get:

Ideal_Philips_Refrig_H2(P_min, P_max, TH, TL) =

"P1(bar)"	"P2 (bar)"	"P3 (bar)"	"P4 (bar)"	"Qa_by_m(kJ/kg)"	"W_net_by_m(kJ/kg)"	"COP"	"COP_i"	"FOM"
3.039	10.13	3.377	1.013	493.146	999.197	0.494	0.5	0.987

Thus:

Refrign. effect: Qa_by_m = 493.146 **kJ/kg .... Ans.**

COP: COP = 0.494 **...Ans.**

FOM: FOM = 0.987 **....Ans.**

Plot Refrig. effect as max. pressure varies from 5 to 10 bar, other conditions remaining the same:

First write the Refrig. effect as function of P_max, the max. pressure:

Qa_by_m(P_max) := Ideal_Philips_Refrig_H2(P_min, P_max, TH, TL)$_{1,4}$

Now:

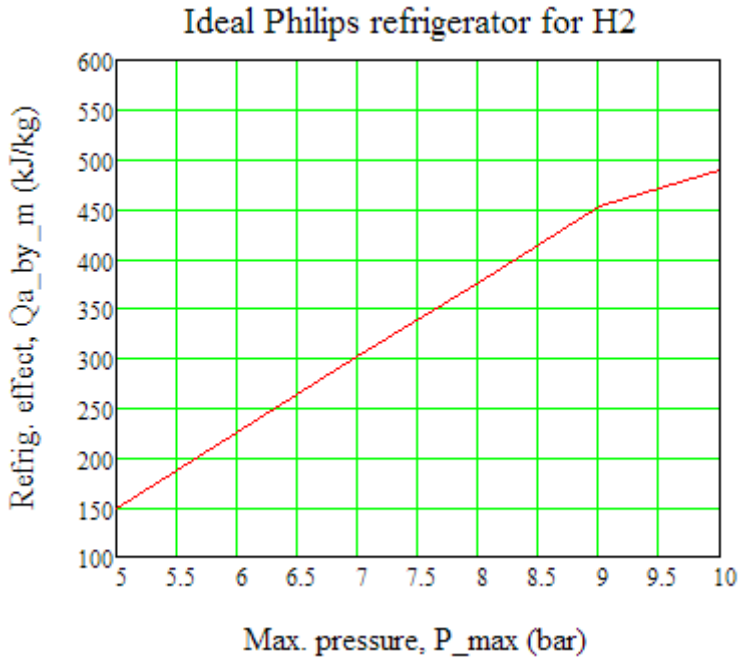
P_max := 5, 6.. 10...define a range variable

And, we get:

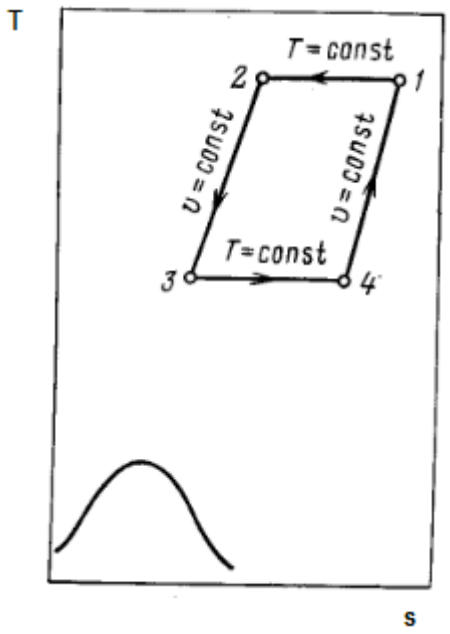
P_max =	Qa_by_m(P_max)
5	149.003
6	224.987
7	300.97
8	376.953
9	452.937
10	488.52

Download free eBooks at bookboon.com

Now, plot the results:



Prob. 4.2.25 Determine the fraction of the ideal refrigeration effect (Δ Q/Q_ideal) that is lost in a Philips refrigerator having a regenerator which is 98% efficient. Hydrogen gas is the working substance (cp/cv = 1.4), and the volume ratio (v4/v3) is 1.85. The refrigerator operates between the temp limits of 300 K and 80 K. [1]



Download free eBooks at bookboon.com

Data:

$\varepsilon := 0.98$ ...regen. effectiveness $\gamma := 1.4$ ...(cp/cv) for H2

$T2 := 300$ K ... sink temp $T3 := 80$ K .... ssource temp

$v4\_by\_v3 := 1.85$ ...volume ratio

Then, we have [1]:

$$\text{Fraction} = \left(\frac{\Delta Q}{Qa\_ideal} = \frac{1-\varepsilon}{\gamma - 1} \cdot \frac{\frac{T2}{T3} - 1}{\ln\left(\frac{v4}{v3}\right)} \right)$$

i.e. $$\text{Fraction} := \frac{1-\varepsilon}{\gamma - 1} \cdot \frac{\frac{T2}{T3} - 1}{\ln(v4\_by\_v3)}$$

i.e. $\text{Fraction} = 0.224 = \frac{\Delta Q}{Qa\_ideal}$ **...fraction of ideal refrign. effect that is lost because the regenerator effectiveness is less than unity.... Ans.**

Brain power
By 2020, wind could provide one-tenth of our planet's electricity needs. Already today, SKF's innovative know-how is crucial to running a large proportion of the world's wind turbines.
Up to 25 % of the generating costs relate to maintenance. These can be reduced dramatically thanks to our systems for on-line condition monitoring and automatic lubrication. We help make it more economical to create cleaner, cheaper energy out of thin air.
By sharing our experience, expertise, and creativity, industries can boost performance beyond expectations.
Therefore we need the best employees who can meet this challenge!
The Power of Knowledge Engineering
Plug into The Power of Knowledge Engineering.
Visit us at www.skf.com/knowledge
SKF

Download free eBooks at bookboon.com

Click on the ad to read more

Plot Fraction as ε varies from 0.91 to 1:

First, write Fraction as a function of ε:

$$\text{Fraction}(\varepsilon) := \frac{1-\varepsilon}{\gamma - 1} \cdot \frac{\frac{T2}{T3} - 1}{\ln(v4\_by\_v3)}$$

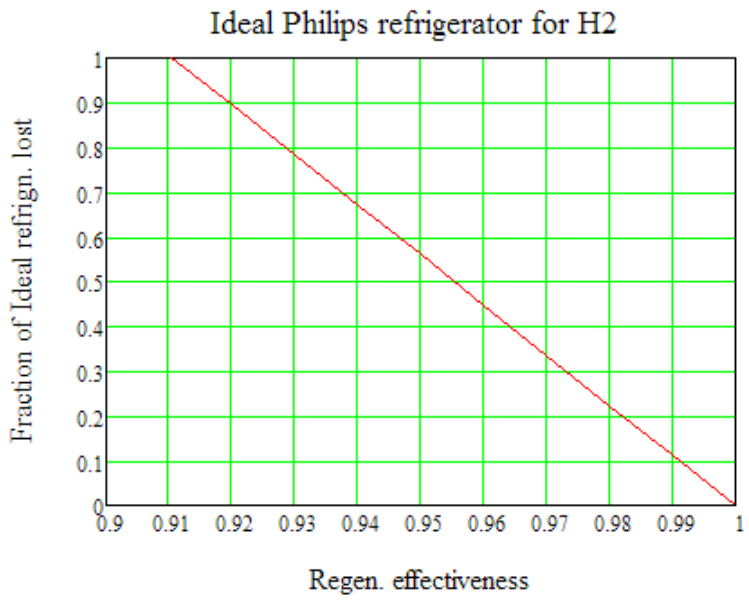
Now:

$\varepsilon := 0.91, 0.92 .. 1$ ....define a range variable

And, we get:

ε =	Fraction(ε) =
0.91	1.006
0.92	0.894
0.93	0.782
0.94	0.671
0.95	0.559
0.96	0.447
0.97	0.335
0.98	0.224
0.99	0.112
1	0

Now, plot the results:



Download free eBooks at bookboon.com

Note: Fraction of refrigeration lost is 1 when effectiveness is 0.91 or less. i.e. if the effectiveness of regenerator falls below 0.91, all the refrigeration produced will be lost.

Schmidt's theory for analysis of Stirling cycle:

See the theory and calculation equations given earlier.

Recollect:

Alpha type Stirling engine:

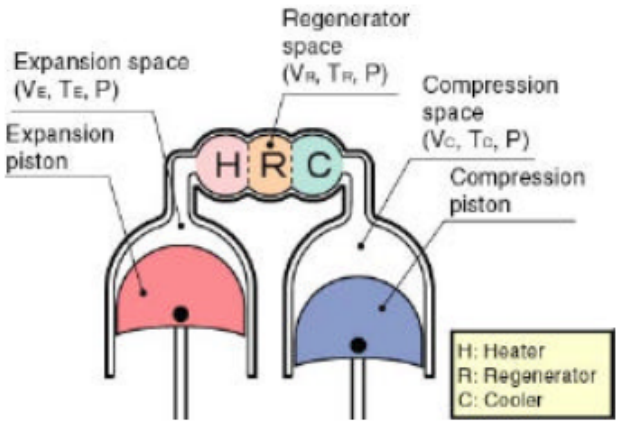


Fig. Alpha-type Stirling Engine

Based on the calculation equations, given earlier, for the Alpha type Stirling engine,

Mathcad Function to plot the P-v diagram and also to calculate other performance parameters is given below:

In the program given below:

Inputs: x (crank angle, deg.), dx (phase angle = 90 deg.), TC (comprn. space gas temp, K), TE(expn. space gas temp, K), VSE (swept vol. of expn. piston or displacer, m^3), VSc (swept vol. of comprn. piston or power piston, m^3), VDE (dead vol of expn. space, m^3), VDC(dead vol of comprn. space, m^3), VR(regenerator volume, m^3), N (rpm), Rgas (gas constant, J/kg.K), Pmean (mean pressure, kPa).

Download free eBooks at bookboon.com

Outputs: In matrix form: x (crank angle, deg.), P (engine pressure, kPa), V (total momental volume, m^3).

VE (expn. space momental vol, m^3), VC (comprn. space momental vol, m^3), LE (indicated expn power, kW), LC (indicated comprn power, kW), Li (indicated power, kW), m (total mass of working gas, kg)

$$\text{SchmidtAnalysis}\left(x, dx, T_C, T_E, V_{SE}, V_{SC}, V_{DE}, V_{DC}, V_R, N, R_{gas}, P_{mean}\right) :=$$

Download free eBooks at bookboon.com
Click on the ad to read more

$$tratio \leftarrow \frac{T_C}{T_E}$$

$$vratio \leftarrow \frac{V_{SC}}{V_{SE}}$$

$$X_{DE} \leftarrow \frac{V_{DE}}{V_{SE}}$$

$$X_{DC} \leftarrow \frac{V_{DC}}{V_{SE}}$$

$$X_R \leftarrow \frac{V_R}{V_{SE}}$$

$$a \leftarrow \operatorname{atan}\left[\frac{vratio \cdot \sin\left(\frac{dx \cdot \pi}{180}\right)}{tratio + \cos\left(\frac{dx \cdot \pi}{180}\right)}\right]$$

$$S \leftarrow tratio + 2 \cdot tratio \cdot X_{DE} + \frac{4 \cdot tratio \cdot X_R}{1 + tratio} + vratio + 2 \cdot X_{DC}$$

$$B \leftarrow \sqrt{tratio^2 + 2 \cdot tratio \cdot vratio \cdot \cos\left(\frac{dx \cdot \pi}{180}\right) + vratio^2}$$

$$c \leftarrow \frac{B}{S}$$

$$m \leftarrow \frac{P_{mean} \cdot V_{SE} \cdot \sqrt{S^2 - B^2}}{2 \cdot R_{gas} \cdot T_C}$$

$$P \leftarrow \frac{P_{mean} \cdot \sqrt{1 - c^2}}{1 - c \cdot \cos\left(\frac{x \cdot \pi}{180} - a\right)}$$

Download free eBooks at bookboon.com

$$V_E \leftarrow \frac{V_{SE}}{2} \cdot \left(1 - \cos\left(\frac{x \cdot \pi}{180}\right)\right) + V_{DE}$$

$$V_C \leftarrow \frac{V_{SC}}{2} \cdot \left(1 - \cos\left(\frac{x - dx}{180} \cdot \pi\right)\right) + V_{DC}$$

$$V \leftarrow V_E + V_R + V_C$$

$$W_E \leftarrow \frac{P_{mean} \cdot V_{SE} \cdot \pi \cdot c \cdot \sin(a)}{1 + \sqrt{1 - c^2}}$$

$$W_C \leftarrow -\frac{P_{mean} \cdot V_{SE} \cdot \pi \cdot c \cdot tratio \cdot \sin(a)}{1 + \sqrt{1 - c^2}}$$

$$W_i \leftarrow W_E + W_C$$

$$L_i \leftarrow \frac{W_i \cdot N}{60}$$

$$L_E \leftarrow \frac{W_E \cdot N}{60}$$

$$L_C \leftarrow \frac{W_C \cdot N}{60}$$

$$\eta \leftarrow \frac{W_i}{W_E}$$

$$\eta_{carnot} \leftarrow 1 - tratio$$

$$P_{min} \leftarrow P_{mean} \cdot \sqrt{\frac{1 - c}{1 + c}}$$

$$P_{max} \leftarrow P_{mean} \cdot \sqrt{\frac{1 + c}{1 - c}}$$

"x (deg.)"	"P(kPa)"	"V(m^3)"	"VE(m^3)"	"VC(m^3)"	"LE(kW)"	"LC(kW)"	"Li(kW)"	"Effcy.(eta)"	"m(kg)"
x	P	V	V_E	V_C	L_E	L_C	L_i	η	m

Example: Draw a P-V diagram and calculate the indicated power of an Alpha-type Stirling engine, with air as the working fluid, under following conditions [19]:

Swept volume of an expansion piston: 0.628 cm3, swept volume of a compression piston: 0.628 cm3, dead volume of the expansion space: 0.2cm3, dead volume of the compression space: 0.2cm3, regenerator volume: 0.2cm3, phase angle: 90deg, mean pressure: 101.3 kPa, expansion gas temperature: 400 deg C, compression gas temperature: 30 deg C, engine speed: 2000 rpm.

Download free eBooks at bookboon.com

Data:

$T_C := 30 + 273$ $T_E := 400 + 273$ K

$V_{SE} := 0.628 \cdot 10^{-6}$ $V_{SC} := 0.628 \cdot 10^{-6}$ $V_{DE} := 0.2 \cdot 10^{-6}$ $V_{DC} := 0.2 \cdot 10^{-6}$ ...m^3

$V_R := 0.2 \cdot 10^{-6}$ ....m^3

$P_{mean} := 101.3$ kPa $N := 2000$ rpm $R_{gas} := 0.287$ kJ/kg.K.....Gas constant for air

$x := 0$ ..., deg,....crank angle

$dx := 90$ ....deg,....phase angle

To draw the P-v diagram:

$x := 0, 10 .. 360$ deg..... define a range variable

Let:

$$Stirling(x) := SchmidtAnalysis\left(x, dx, T_C, T_E, V_{SE}, V_{SC}, V_{DE}, V_{DC}, V_R, N, R_{gas}, P_{mean}\right)$$

LIGS University
based in Hawaii, USA
is currently enrolling in the Interactive Online BBA, MBA, MSc, DBA and PhD programs:
- enroll by October 31st, 2014 and
- save up to 11% on the tuition!
- pay in 10 installments / 2 years
- Interactive Online education
- visit www.ligsuniversity.com to find out more!

Note: LIGS University is not accredited by any nationally recognized accrediting agency listed by the US Secretary of Education.
More info here.

Download free eBooks at bookboon.com
Click on the ad to read more

For crank angle, x = 0 deg:

We get, for ex:

Stirling(0) =

	0	1	2	3	4	5
0	"x (deg.)"	"P(kPa)"	"V(m^3)"	"VE(m^3)"	"VC(m^3)"	"LE(kW)"
1	0	111.077	$9.14 \cdot 10^{-7}$	$2 \cdot 10^{-7}$	$5.14 \cdot 10^{-7}$	$1.254 \cdot 10^{-3}$

6	7	8	9
"LC(kW)"	"Li(kW)"	"Effcy.(eta)"	"m(kg)"
$-5.646 \cdot 10^{-4}$	$6.894 \cdot 10^{-4}$	0.55	$9.302 \cdot 10^{-7}$

For crank angle, x = 20 deg:

Stirling(20) =

	0	1	2	3	4	5
0	"x (deg.)"	"P(kPa)"	"V(m^3)"	"VE(m^3)"	"VC(m^3)"	"LE(kW)"
1	20	128.526	$8.255 \cdot 10^{-7}$	$2.189 \cdot 10^{-7}$	$4.066 \cdot 10^{-7}$	$1.254 \cdot 10^{-3}$

6	7	8	9
"LC(kW)"	"Li(kW)"	"Effcy.(eta)"	"m(kg)"
$-5.646 \cdot 10^{-4}$	$6.894 \cdot 10^{-4}$	0.55	$9.302 \cdot 10^{-7}$

And, to draw the P-v diagram, we have:

$\text{Stirling}(20)_{1,1} = 128.526$ = P (kPa)

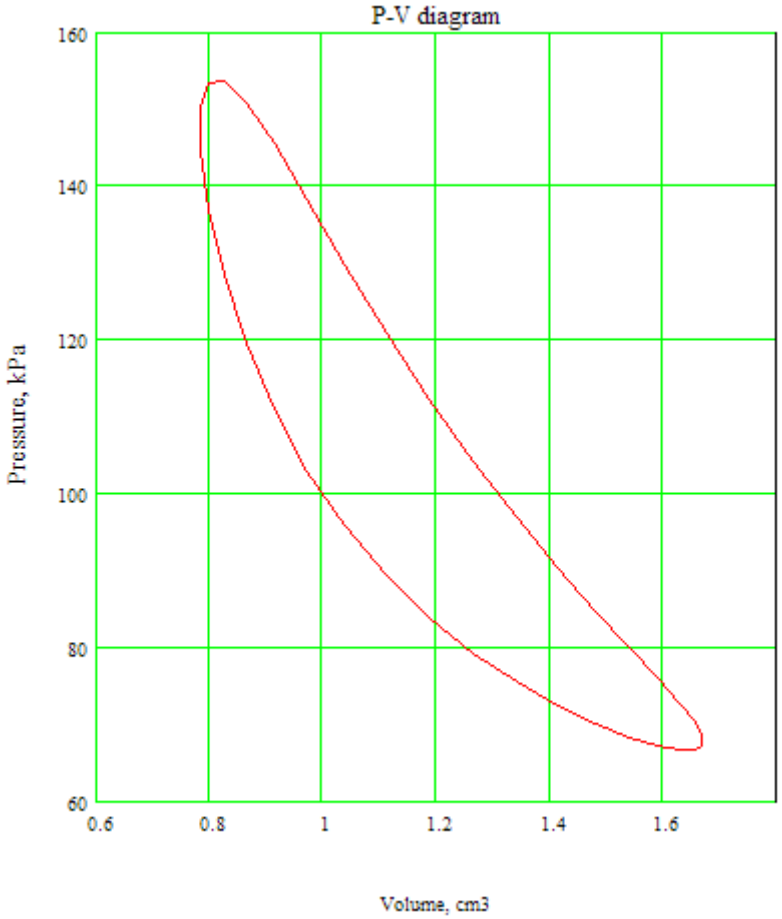
$\text{Stirling}(20)_{1,2} = 8.255 \cdot 10^{-7}$ = V (m^3)

We get:

x (deg.)	P (kPa)	V (cm^3)	x (deg.)	P (kPa)	V (cm^3)
0	111.077	0.914	190	76.07	1.592
10	119.685	0.864	200	72.884	1.63
20	128.526	0.826	210	70.398	1.657
30	137.061	0.799	220	68.566	1.67
40	144.583	0.786	230	67.351	1.67
50	150.299	0.786	240	66.728	1.657
60	153.496	0.799	250	66.684	1.63
70	153.728	0.826	260	67.218	1.592
80	150.964	0.864	270	68.342	1.542
90	145.59	0.914	280	70.078	1.483
100	138.292	0.973	290	72.461	1.416
110	129.862	1.04	300	75.538	1.343
120	121.028	1.113	310	79.36	1.267
130	112.357	1.189	320	83.985	1.189
140	104.231	1.267	330	89.462	1.113
150	96.87	1.343	340	95.823	1.04
160	90.374	1.416	350	103.056	0.973
170	84.763	1.483	360	111.077	0.914
180	80.011	1.542			

Download free eBooks at bookboon.com

And, plot:



Also, plot the total volume (V), expansion volume (VE) and compression volume (VC) against the crank angle, x:

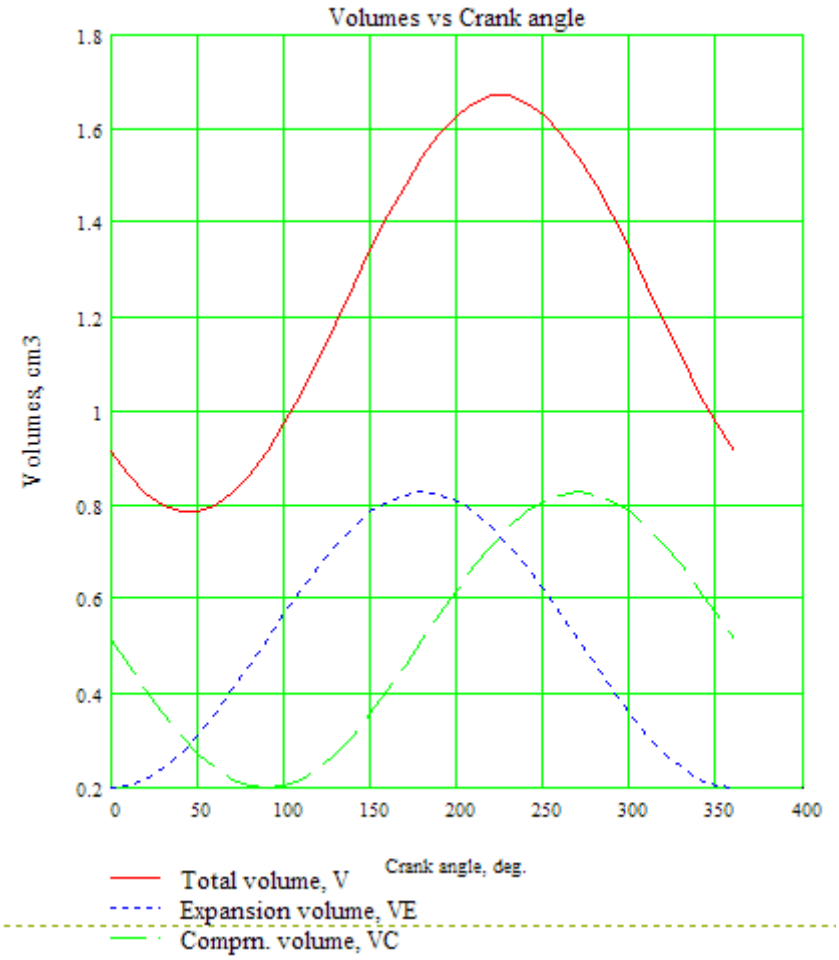
We have:

$V(x) := Stirling(x)_{1,2} \cdot 10^6$ cm^3 $V_E(x) := Stirling(x)_{1,3} \cdot 10^6$ cm^3

$V_C(x) := Stirling(x)_{1,4} \cdot 10^6$ cm^3

Download free eBooks at bookboon.com

Plot:



To draw the plot of Indicated power, Li vs N:

$N := 1000, 1100 .. 2000$ deg..... define a range variable

Let:

$$Stirling(N) := SchmidtAnalysis(x, dx, T_C, T_E, V_{SE}, V_{SC}, V_{DE}, V_{DC}, V_R, N, R_{gas}, P_{mean})$$

Then, we get:

$Li(N) := Stirling(N)_{1,7}$ i.e. $Li(1000) = 3.447 \times 10^{-4}$ kW

Download free eBooks at bookboon.com

We get:

N =	Li(N) =
$1 \cdot 10^3$	$3.447 \cdot 10^{-4}$
$1.1 \cdot 10^3$	$3.792 \cdot 10^{-4}$
$1.2 \cdot 10^3$	$4.137 \cdot 10^{-4}$
$1.3 \cdot 10^3$	$4.481 \cdot 10^{-4}$
$1.4 \cdot 10^3$	$4.826 \cdot 10^{-4}$
$1.5 \cdot 10^3$	$5.171 \cdot 10^{-4}$
$1.6 \cdot 10^3$	$5.515 \cdot 10^{-4}$
$1.7 \cdot 10^3$	$5.86 \cdot 10^{-4}$
$1.8 \cdot 10^3$	$6.205 \cdot 10^{-4}$
$1.9 \cdot 10^3$	$6.55 \cdot 10^{-4}$
$2 \cdot 10^3$	$6.894 \cdot 10^{-4}$

TURN TO THE EXPERTS FOR
SUBSCRIPTION CONSULTANCY

Subscrybe is one of the leading companies in Europe when it comes to innovation and business development within subscription businesses.

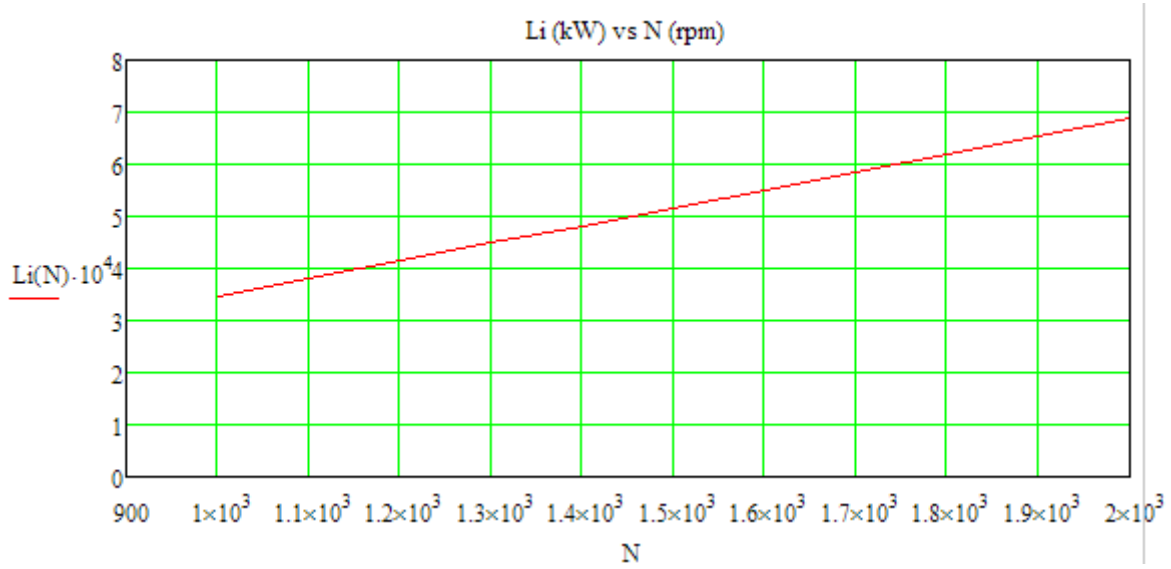
We innovate new subscription business models or improve existing ones. We do business reviews of existing subscription businesses and we develope acquisition and retention strategies.

Learn more at linkedin.com/company/subscrybe or contact
Managing Director Morten Suhr Hansen at mha@subscrybe.dk

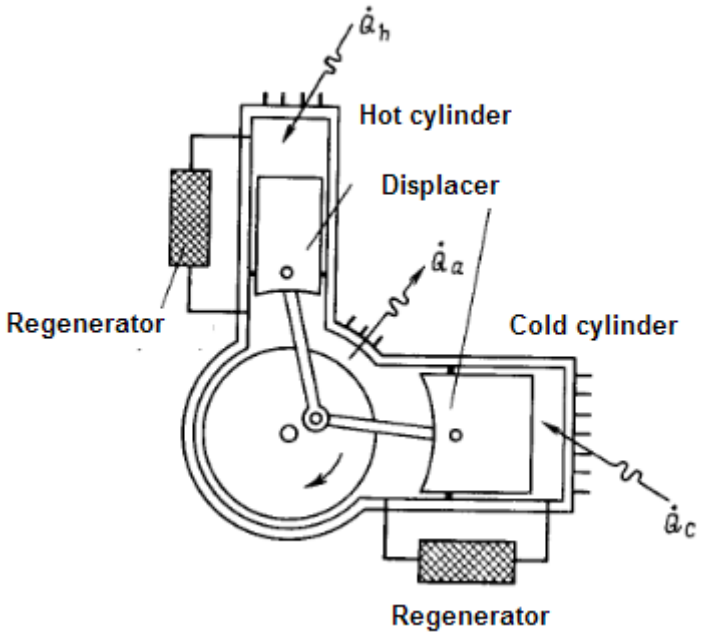
SUBSCRYBE - to the future

Download free eBooks at bookboon.com
Click on the ad to read more

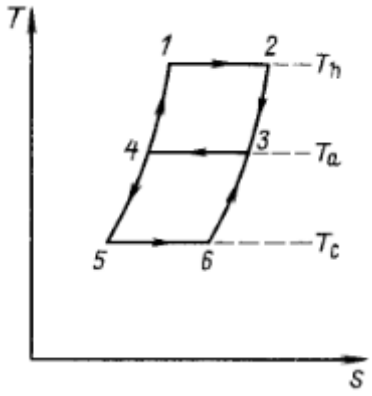
And, plot:



Prob. 4.2.26 An ideal Vuilleumier refrigerator absorbs energy as heat from a high temp source at 600 K and rejects heat to a sink at 300 K. The refrigerator absorbs heat from the low temp region at 20 K. If the heat transfer rate at the hot end of the refrigerator is 2 kW, determine the refrigeration rate at the cold end. [1]



Download free eBooks at bookboon.com



Data:

$T_h := 600$ K $T_a := 300$ K $T_c := 20$ K $Q_h := 2$ kW

Then, we have:

$$COP = \frac{Q_c}{Q_h} = \frac{T_c}{T_h} \cdot \frac{(T_h - T_a)}{(T_a - T_c)}$$

"I studied English for 16 years but... ...I finally learned to speak it in just six lessons"
Jane, Chinese architect
ENGLISH OUT THERE
Click to hear me talking before and after my unique course download

Download free eBooks at bookboon.com
Click on the ad to read more

i.e. $$\frac{Q_c}{Q_h} = \frac{T_c}{T_h}\cdot\frac{(T_h - T_a)}{(T_a - T_c)} = 0.0357$$

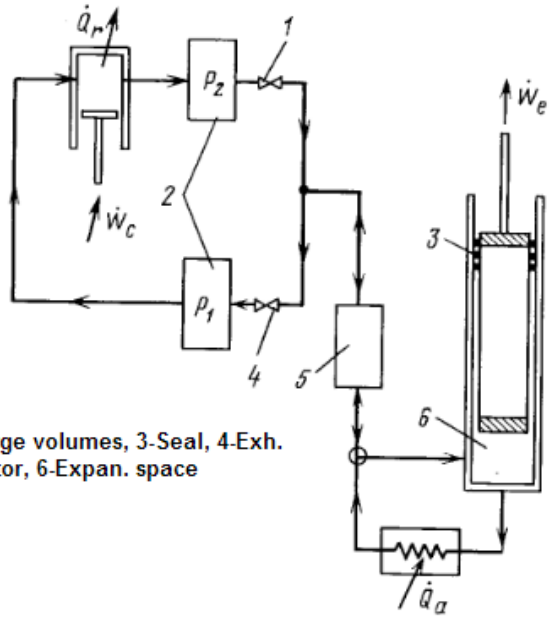
Therefore: $$Q_c := Q_h\cdot\left[\frac{T_c}{T_h}\cdot\frac{(T_h - T_a)}{(T_a - T_c)}\right]$$

i.e. $Q_c = 0.0714$ **kW = 71.4 W .... Ans.**

==========

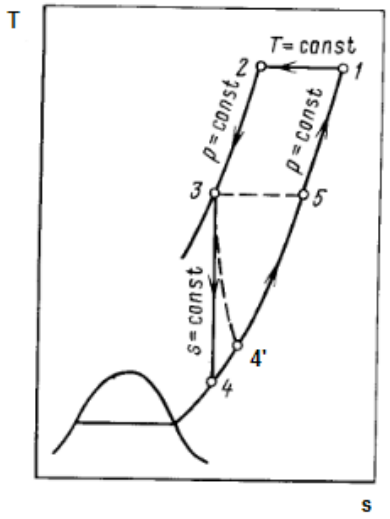
Prob.4.2.27 A Solvay refrigerator uses neon as the working fluid, with the expander work utilized to aid in compression. Neon is compressed from 152 kPa to 1.013 MPa and 294 K in a compressor having an overall isothermal effcy of 75%. The adiabatic effcy of the expander is 90% and the mechanical effcy of expander is 96%. At the beginning of the expansion process, the temp of neon gas is 85.6 K and the pressure is 1.013 MPa. The neon gas enters the regenerator at the cold end at 152 kPa and 85.6 K. Determine the COP of the refrigerator. [1]

Solvay refrigerator.



1-Inlet valve, 2-Surge volumes, 3-Seal, 4-Exh. valve, 5-Regenerator, 6-Expan. space

T-s diagram



Data:

P1 := 1.52 bar P2 := 10.13 bar T1 := 294 K T2 := T1 P3 := P2

P5 := P1 T3 := 85.6 K T5 := T3

Download free eBooks at bookboon.com

eta_co := 0.75 ...overall isoth. effcy of compressor

eta_ad := 0.9 ...adiabatic effcy of expander

eta_e_m := 0.96 ...mechanical effcy of expander

Then, we have:

h1 := enthalpy_Ne(P1, T1) i.e. h1 = 362.518 kJ/kg

s1 := entropy_Ne(P1, T1) i.e. s1 = 5.522 kJ/kg.K

h2 := enthalpy_Ne(P2, T2) i.e. h2 = 362.775 kJ/kg

s2 := entropy_Ne(P2, T2) i.e. s2 = 4.716 kJ/kg.K

h3 := enthalpy_Ne(P3, T3) i.e. h3 = 145.556 kJ/kg

s3 := entropy_Ne(P3, T3) i.e. s3 = 3.424 kJ/kg.K

s4 := s3 ...for isentropic expansion

h4 := enthalpy_Ne_Ps(P1, s4) i.e. h4 = 99.053 kJ/kg

h5 := enthalpy_Ne(P5, T5) i.e. h5 = 147.462 kJ/kg

Then, we get:

Refrig. effect:

h4_prime := h3 − eta_ad·(h3 − h4) i.e. h4_prime = 103.703 kJ/kg

And, Qa_by_m := h5 − h4_prime

i.e. Qa_by_m = 43.759 **kJ/kg .... refrig. effect .... Ans.**

Net compressor work:

$$Wnet\_by\_m := \frac{[T2\cdot(s1 - s2) - (h1 - h2)]}{eta\_co} - eta\_e\_m\cdot eta\_ad\cdot(h3 - h4)$$

i.e. Wnet_by_m = 276.091 **kJ/kg ..... net compressor work .... Ans.**

Download free eBooks at bookboon.com

Coeff. of Performance:

$$COP := \frac{Qa\_by\_m}{Wnet\_by\_m}$$

i.e. COP = 0.158 ....Ans.

Plot Refrig. effect and COP as high pressure P2 varies from 5 to 20 bar:

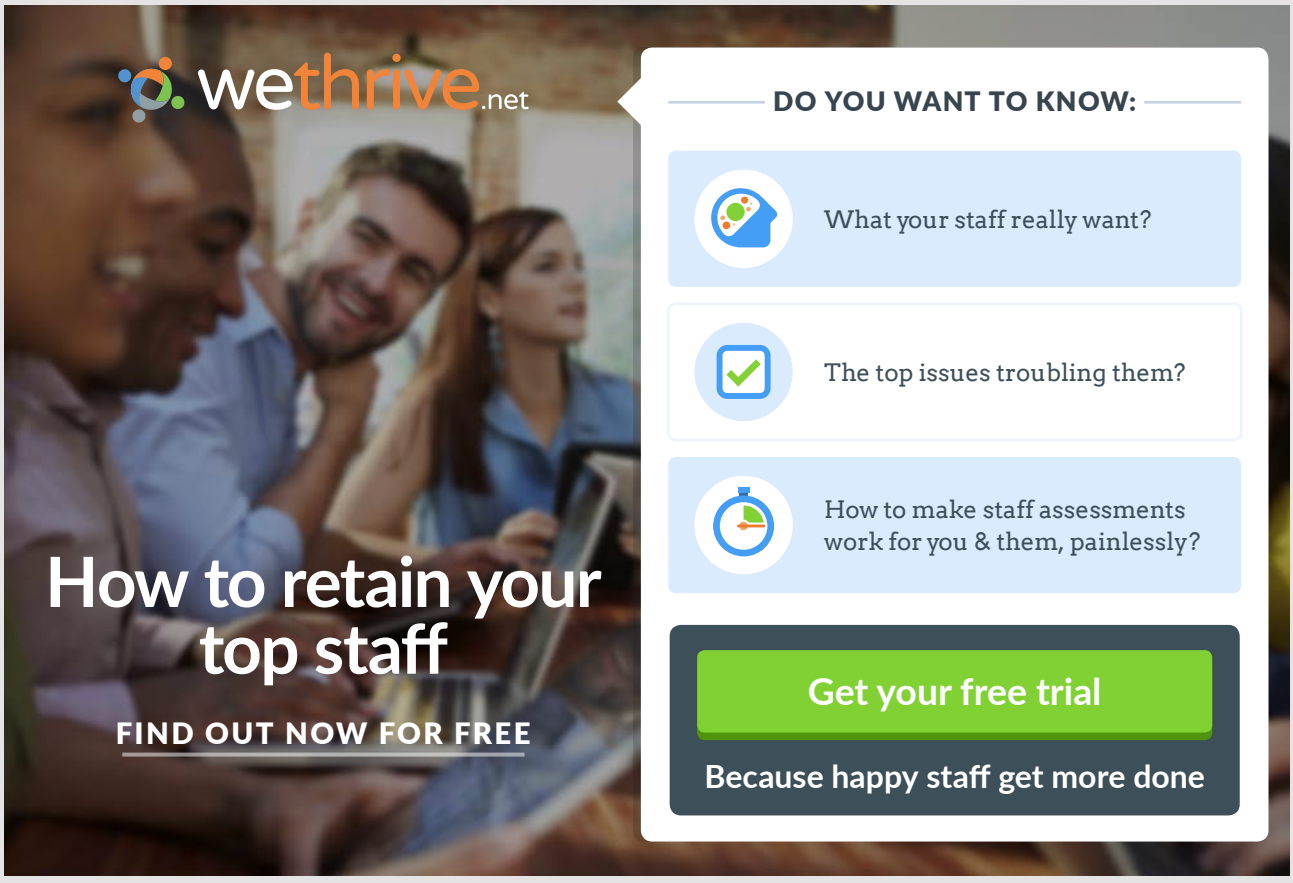
First write relevant quantities as functions of P2:

h2(P2) := enthalpy_Ne(P2, T2)

s2(P2) := entropy_Ne(P2, T2)

h3(P2) := enthalpy_Ne(P2, T3)

s3(P2) := entropy_Ne(P2, T3)


wethrive.net
How to retain your top staff
FIND OUT NOW FOR FREE
DO YOU WANT TO KNOW:
What your staff really want?
The top issues troubling them?
How to make staff assessments work for you & them, painlessly?
Get your free trial
Because happy staff get more done

Download free eBooks at bookboon.com
Click on the ad to read more

$s4(P2) := s3(P2)$ ...for isentropic expansion

$h4(P2) := enthalpy\_Ne\_Ps(P1, s4(P2))$

$h4\_prime(P2) := h3(P2) - eta\_ad \cdot (h3(P2) - h4(P2))$

And, $Qa\_by\_m(P2) := h5 - h4\_prime(P2)$

$$Wnet\_by\_m(P2) := \frac{[T2 \cdot (s1 - s2(P2)) - (h1 - h2(P2))]}{eta\_co} - eta\_e\_m \cdot eta\_ad \cdot (h3(P2) - h4(P2))$$

$$COP(P2) := \frac{Qa\_by\_m(P2)}{Wnet\_by\_m(P2)}$$

Now:

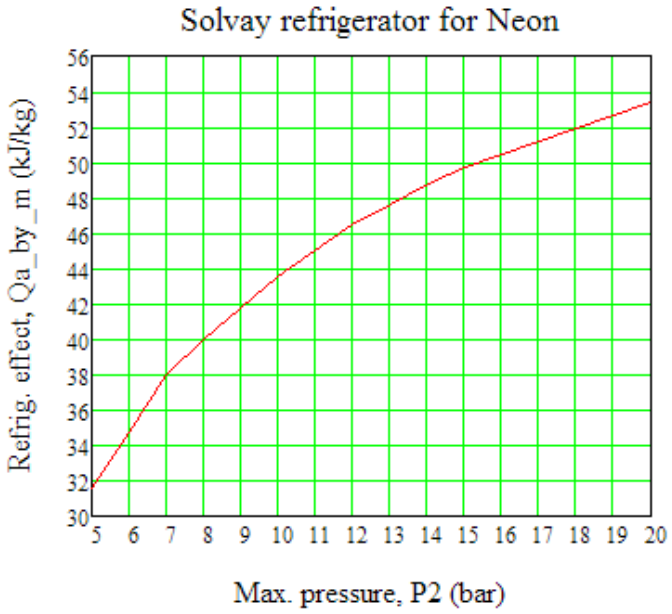
$P2 := 5, 6 .. 20$ ...define a range variable

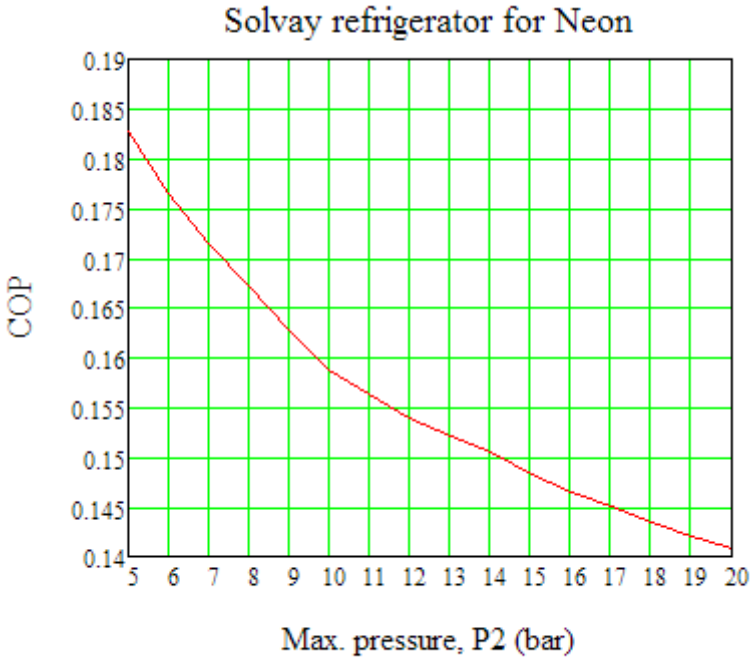
And, we get:

P2 =	Qa_by_m(P2)	Wnet_by_m(P2)	COP(P2) =
5	31.533	172.464	0.183
6	34.728	196.853	0.176
7	37.924	221.242	0.171
8	39.919	238.827	0.167
9	41.746	256.572	0.163
10	43.574	274.318	0.159
11	44.997	287.958	0.156
12	46.419	301.597	0.154
13	47.598	312.764	0.152
14	48.75	323.955	0.15
15	49.692	335.348	0.148
16	50.438	344.212	0.147
17	51.184	353.077	0.145
18	51.93	361.941	0.143
19	52.677	370.805	0.142
20	53.423	379.67	0.141

Download free eBooks at bookboon.com

And, plot the results:



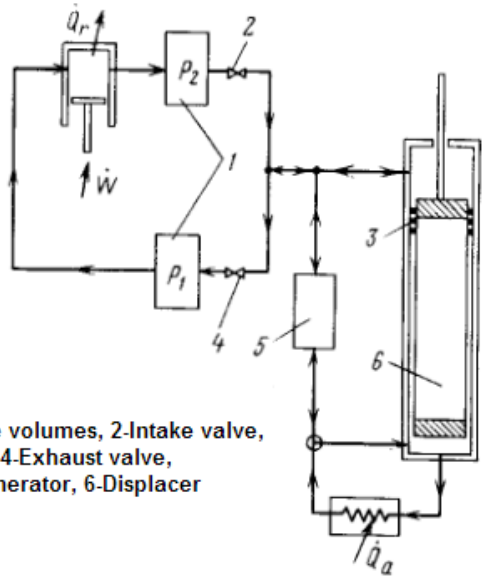


==========

Download free eBooks at bookboon.com

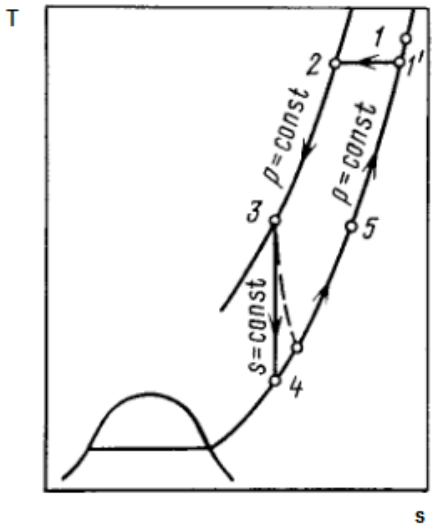
Prob. 4.2.28 Write a Mathcad program to find the various parameters for a Gifford-McMahon (G-M) cycle refrigerator, using Helium as the working gas.

G-M cycle refrigerator.



1-Surge volumes, 2-Intake valve,
3-Seal, 4-Exhaust valve,
5-Regenerator, 6-Displacer

T-s diagram



Download free eBooks at bookboon.com

Click on the ad to read more

In the Mathcad Function that we are going to write, we need the specific volume of helium to be calculated. Reference [20] gives Mathcad Functions for properties of Helium (and other fluids). We add below, Functions to determine sp. volume and internal energy, using the data table generated from the NIST website:

We have, the vector containing pressures for which data tables have been generated from NIST website:

$$Pressures\_He := (1\ \ 2\ \ 5\ \ 10\ \ 15\ \ 20\ \ 30\ \ 40\ \ 50\ \ 60\ \ 70\ \ 80\ \ 90\ \ 100\ \ 110\ \ 120\ \ 130\ \ 140\ \ 150\ \ 160\ \ 170\ \ 180\ \ 190\ \ 200\ \ 210\ \ 220)\ \ \text{...bar}$$

Now, write the Function for sp. volume, v and internal energy, u:

$$\begin{aligned}
&v_and_u_He(P,T) := \\
&\quad \text{return "T < Tsat; use two-phase Functions" if } T < TSAT_He(P) \wedge P < 2.2746 \\
&\quad \text{return "P should be between 1 bar and 220 bar" if } P < 1 \vee P > 220 \\
&\quad \text{for } k \in 0..25 \\
&\qquad \text{if } P = Pressures_He_{0,k} \\
&\qquad\quad v \leftarrow \text{linterp}\left[\left(M_He_{Pressures_He_{0,k}}\right)^{\langle 0\rangle}, \left(M_He_{Pressures_He_{0,k}}\right)^{\langle 1\rangle}, T\right] \\
&\qquad\quad u \leftarrow \text{linterp}\left[\left(M_He_{Pressures_He_{0,k}}\right)^{\langle 0\rangle}, \left(M_He_{Pressures_He_{0,k}}\right)^{\langle 2\rangle}, T\right] \\
&\qquad\quad \text{return } \begin{pmatrix} \text{"Sp. vol. (m\^3/kg)"} & \text{"Int. energy (kJ/kg)"} \\ v & u \end{pmatrix} \\
&\qquad \text{if } P > Pressures_He_{0,k} \wedge P < Pressures_He_{0,k+1} \\
&\qquad\quad PL \leftarrow Pressures_He_{0,k} \\
&\qquad\quad PH \leftarrow Pressures_He_{0,k+1} \\
&\qquad\quad A \leftarrow \text{linterp}\left[\left(M_He_{PL}\right)^{\langle 0\rangle}, \left(M_He_{PL}\right)^{\langle 1\rangle}, T\right] \\
&\qquad\quad B \leftarrow \text{linterp}\left[\left(M_He_{PH}\right)^{\langle 0\rangle}, \left(M_He_{PH}\right)^{\langle 1\rangle}, T\right] \\
&\qquad\quad v \leftarrow A + \left(\frac{P - PL}{PH - PL}\right)\cdot(B - A) \\
&\qquad\quad C \leftarrow \text{linterp}\left[\left(M_He_{PL}\right)^{\langle 0\rangle}, \left(M_He_{PL}\right)^{\langle 2\rangle}, T\right] \\
&\qquad\quad D \leftarrow \text{linterp}\left[\left(M_He_{PH}\right)^{\langle 0\rangle}, \left(M_He_{PH}\right)^{\langle 2\rangle}, T\right] \\
&\qquad\quad u \leftarrow C + \left(\frac{P - PL}{PH - PL}\right)\cdot(D - C) \\
&\qquad\quad \text{return } \begin{pmatrix} \text{"Sp. vol. (m\^3/kg)"} & \text{"Int. energy (kJ/kg)"} \\ v & u \end{pmatrix}
\end{aligned}$$

Download free eBooks at bookboon.com

Ex: $P := 220$ bar $T := 355$ K

$$v\_and\_u\_He(P,T) = \begin{pmatrix} \text{"Sp. vol. (m^3/kg)"} & \text{"Int. energy (kJ/kg)"} \\ 0.0363 & 1.1194 \times 10^{3} \end{pmatrix}$$

And, from the above, we write:

```
Sp_vol_He(P,T) := return "P should be between 1 bar and 220 bar"  if P < 1 ∨ P > 220
                  return "T should be between 5 K and 355 K"  if T < 5 ∨ T > 355
                  tsat ← TSAT_He(P)  if P ≤ 2.2746
                  v ← v_and_u_He(P,T)[1,0]  if (T ≥ tsat ∧ P ≤ 2.2746)
                  v ← v_and_u_He(P,T)[1,0]  if (P > 2.2746)
                  (return "State point in two phase region--- use 2 phase Functions")  if (T < tsat ∧ P < 2.2746)
```

Ex: $P := 10$ bar $T := 5$ K

$Sp\_vol\_He(P,T) = 6.9787 \times 10^{-3}$ m^3/kg

```
Int_energy_He(P,T) := return "P should be between 1 bar and 220 bar"  if P < 1 ∨ P > 220
                      return "T should be between 5 K and 355 K"  if T < 5 ∨ T > 355
                      tsat ← TSAT_He(P)  if P ≤ 2.2746
                      u ← v_and_u_He(P,T)[1,1]  if (T ≥ tsat ∧ P ≤ 2.2746)
                      u ← v_and_u_He(P,T)[1,1]  if (P > 2.2746)
                      (return "State point in two phase region--- use 2 phase Functions")  if (T < tsat ∧ P < 2.2746)
```

Ex: $P := 10$ bar $T := 5$ K $TSAT\_He(10) = 9.721$

$Int\_energy\_He(P,T) = -0.5349$ kJ/kg

==========

Download free eBooks at bookboon.com

Now, write the desired Function for GM cycle:

Mathcad Function:

GMcycle_Refrig_He(P1, P2, T2, T5, eta_co, eta_ad, epsilon_regen) :=

h2 ← enthalpy_He(P2, T2)
s2 ← entropy_He(P2, T2)
h5 ← enthalpy_He(P1, T5)
T3 ← T5
h3 ← enthalpy_He(P2, T3)
s3 ← entropy_He(P2, T3)
s4 ← s3
h4 ← enthalpy_He_Ps(P1, s4)
T4 ← Temp_He(P1, h4)
h4_prime ← h4 + (1 − eta_ad)·(h3 − h4)
T4_prime ← Temp_He(P1, h4_prime)
v3 ← Sp_vol_He(P2, T3)
v4_prime ← Sp_vol_He(P1, T4_prime)
DELTA_Q ← (1 − epsilon_regen)·(h2 − h3)

This e-book
is made with
SetaPDF
SETASIGN
PDF components for PHP developers
www.setasign.com
Download free eBooks at bookboon.com
Click on the ad to read more

$$Qa\_by\_m \leftarrow \frac{v3}{v4\_prime}\cdot(h5 - h4\_prime) - DELTA\_Q$$

$$h1\_prime \leftarrow h2 + Qa\_by\_m$$

$$T1\_prime \leftarrow Temp\_He(P1, h1\_prime)$$

$$h1 \leftarrow enthalpy\_He(P1, T2)$$

$$s1 \leftarrow entropy\_He(P1, T2)$$

$$W\_by\_m \leftarrow \frac{T2\cdot(s1 - s2) - (h1 - h2)}{eta\_co}$$

$$COP \leftarrow \frac{Qa\_by\_m}{W\_by\_m}$$

$$COP\_i \leftarrow \frac{T5 - T4\_prime}{T2\cdot\ln\left(\frac{T5}{T4\_prime}\right) - (T5 - T4\_prime)}$$

$$FOM \leftarrow \frac{COP}{COP\_i}$$

"T1_prime (K)"	"T3 (K)"	"T4_prime (K)"	"v3 (m^3/kg)"	"v4_prime (m^3/kg)"	"Qa_by_m (kJ/kg)"	"W_by_m (kJ/kg)"	"COP"	"COP_i"	"FOM"
T1_prime	T3	T4_prime	v3	v4_prime	Qa_by_m	W_by_m	COP	COP_i	FOM

In the above program, we have:

Inputs:

P1, P2 … low and high pressures

T2 … temp at exit of after cooler of compressor; T5 … refrigeration temp

eta_co … adiabatic efficiency of compressor; eta_ad … adiabatic efficiency in expansion space

epsilon_regen … regenerator efficiency

Outputs:

Outputs are presented in a matrix, giving values for:

T1_prime … temp of gas exiting the refrigerator

T3 … temp of high pressure gas at the exit of regenerator in to the expansion space

T4_prime … temp of gas after expansion in the expansion space, taking in to account adiabatic effcy of expansion

Download free eBooks at bookboon.com

v3 … sp. vol. at state 3; v4_prime … sp. vol. at state 4_prime

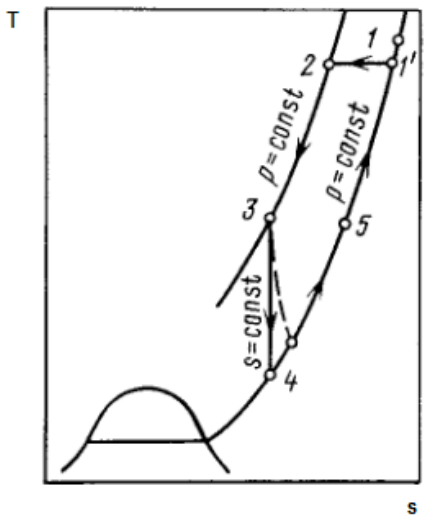
Qa_by_m … refrig effect per kg gas compressed; W_by_m …work done per kg of gas compressed

COP … coeff of performance; COP_i … ideal COP for refrign at constant pressure

FOM … figure of merit for the cycle

Prob. 4.2.29 A G-M refrigerator operates between the pressure limits of 1.013 bar and 10.13 bar using helium as working medium. The max. temp in the space to be cooled is 70 K and the temp of the gas leaving the compressor is 300 K. Assume that regenerator efficiency is 100%, compressor overall efficiency = 60%, adiabatic efficiency of expansion in expansion space = 90%. Determine the refrigeration effect, the COP and the FOM for the system. [1]. Also, plot:

a) refrig. effect vs regen. effcy for T5 = 70, 100 and 120 K
b) the effect on COP and FOM by varying regen. effectiveness, for compressor efficiencies of 0.6, 0.8 and 1.



Data:

P1 := 1.013 bar P2 := 10.13 bar T2 := 300 K T5 := 70 K

eta_co := 0.6 eta_ad := 0.9 epsilon_regen := 1

Download free eBooks at bookboon.com

Using the Function written above, we get:

GMcycle_Refrig_He(P1, P2, T2, T5, eta_co, eta_ad, epsilon_regen) =

	0	1	2	3	4
0	"T1_prime (K)"	"T3 (K)"	"T4_prime (K)"	"v3 (m^3/kg)"	"v4_prime (m^3/kg)"
1	308.8513	70	32.3622	0.147	0.6695

	5	6	7	8	9
	"Qa_by_m (kJ/kg)"	"W_by_m (kJ/kg)"	"COP"	"COP_i"	"FOM"
	43.0012	$2.3972 \cdot 10^3$	0.0179	0.1942	0.0924

i.e. we get:

Qa_by_m(eta_co, epsilon_regen, T5) := GMcycle_Refrig_He(P1, P2, T2, T5, eta_co, eta_ad, epsilon_regen)$_{1,5}$

i.e. Qa_by_m(eta_co, epsilon_regen, T5) = 43.001 kJ/kg ... Ans.

COP(eta_co, epsilon_regen, T5) := GMcycle_Refrig_He(P1, P2, T2, T5, eta_co, eta_ad, epsilon_regen)$_{1,7}$

i.e. COP(eta_co, epsilon_regen, T5) = 0.018 ...Ans.

Deloitte.

Discover the truth at www.deloitte.ca/careers

© Deloitte & Touche LLP and affiliated entities.

Download free eBooks at bookboon.com

Click on the ad to read more

FOM(eta_co, epsilon_regen, T5) := GMcycle_Refrig_He(P1, P2, T2, T5, eta_co, eta_ad, epsilon_regen)$_{1,9}$

i.e. FOM(eta_co, epsilon_regen, T5) = 0.092 ...Ans.

To study the effect on refrig. effect, COP and FOM by varying regen. effectiveness, for compressor efficiencies of 0.6, 0.8 and 1:

We take:

epsilon_regen := 0.96, 0.965.. 1 ....define a range variable

Then, for compressor efficiencies of 0.6, 0.8 and 1, we get:

Refrign. effect:

Qa_by_m(0.6, epsilon_regen, T5)
-4.816
1.161
7.138
13.115
19.092
25.07
31.047
37.024
43.001

Qa_by_m(0.8, epsilon_regen, T5)
-4.816
1.161
7.138
13.115
19.092
25.07
31.047
37.024
43.001

Qa_by_m(1, epsilon_regen, T5)
-4.816
1.161
7.138
13.115
19.092
25.07
31.047
37.024
43.001

COP:

COP(0.6, epsilon_regen, T5)
$-2.009 \cdot 10^{-3}$
$4.842 \cdot 10^{-4}$
$2.978 \cdot 10^{-3}$
$5.471 \cdot 10^{-3}$
$7.964 \cdot 10^{-3}$
0.01
0.013
0.015
0.018

COP(0.8, epsilon_regen, T5)
$-2.679 \cdot 10^{-3}$
$6.456 \cdot 10^{-4}$
$3.97 \cdot 10^{-3}$
$7.295 \cdot 10^{-3}$
0.011
0.014
0.017
0.021
0.024

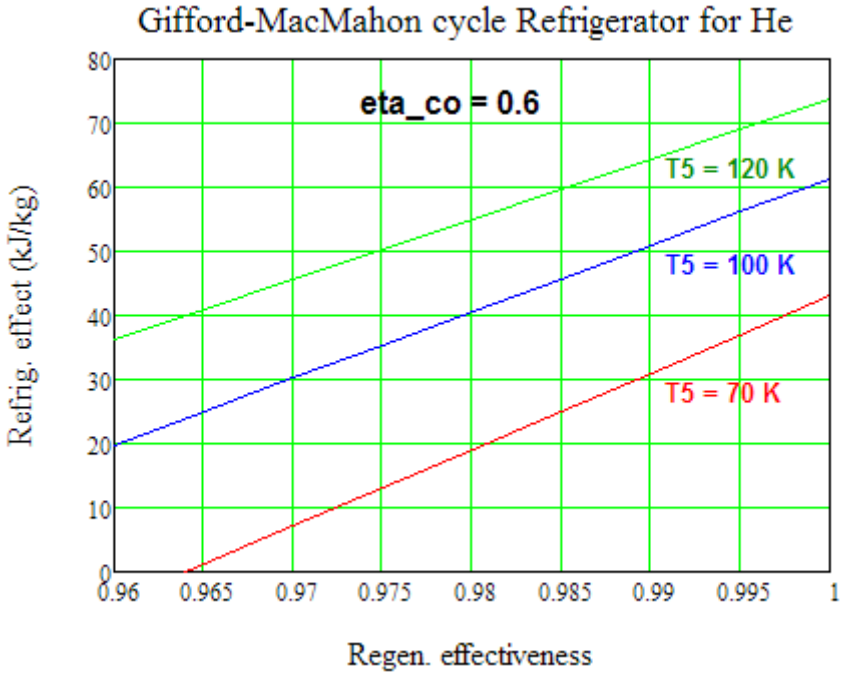
COP(1, epsilon_regen, T5)
$-3.349 \cdot 10^{-3}$
$8.07 \cdot 10^{-4}$
$4.963 \cdot 10^{-3}$
$9.118 \cdot 10^{-3}$
0.013
0.017
0.022
0.026
0.03

Download free eBooks at bookboon.com

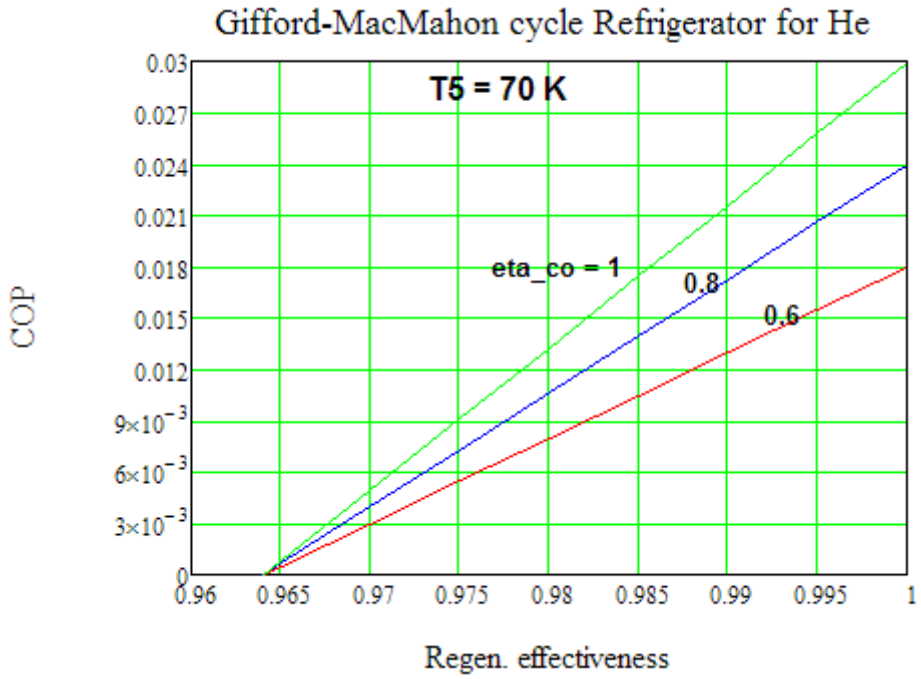
FOM:

FOM(0.6, epsilon_regen, T5)	FOM(0.8, epsilon_regen, T5)	FOM(1, epsilon_regen, T5)
-0.01	-0.014	-0.017
$2.493 \cdot 10^{-3}$	$3.324 \cdot 10^{-3}$	$4.156 \cdot 10^{-3}$
0.015	0.02	0.026
0.028	0.038	0.047
0.041	0.055	0.068
0.054	0.072	0.09
0.067	0.089	0.111
0.08	0.106	0.133
0.092	0.123	0.154

And, the plots:

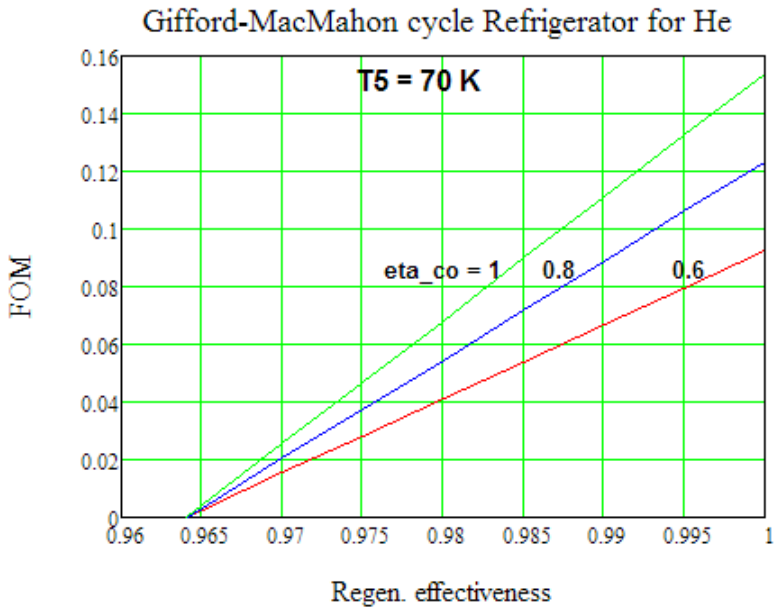


Download free eBooks at bookboon.com

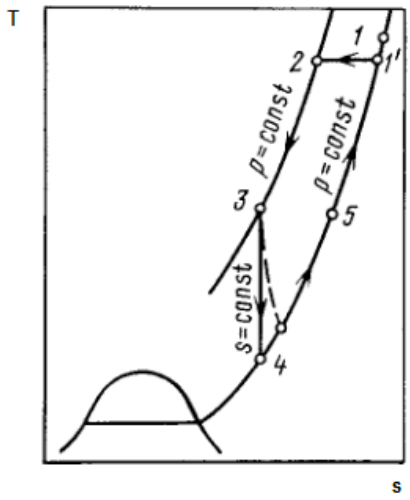


be
your degree
© 2013 Accenture. All rights reserved.
Bring your talent and passion to a global organization at the forefront of business, technology and innovation. Discover how great you can be.
Visit accenture.com/bookboon
Be greater than.
consulting | technology | outsourcing
accenture
High performance. Delivered.

Download free eBooks at bookboon.com
Click on the ad to read more



Prob. 4.2.30 A G-M refrigerator operates between the pressure limits of 1.013 bar (1 atm.), 300 K and 5.066 bar (5 atm.), using helium as working medium. The temp of gas at the beginning of the expansion process is 80 K and the expansion is reversible and adiabatic. The compressor and the regenerator may be assumed to be ideal. Determine the refrigeration effect, and the COP of the refrigerator. [1]Also, plot these quantities as the compressor discharge pressure varies from 4 bar to 10 bar, for refrign. temp. T5 = 80, 100 and 120 K.



Data:

P1 := 1.013 bar P2 := 5.066 bar T2 := 300 K T5 := 80 K

eta_co := 1 eta_ad := 1 epsilon_regen := 1

Download free eBooks at bookboon.com

Using the Function written above, we get:

GMcycle_Refrig_He(P1, P2, T2, T5, eta_co, eta_ad, epsilon_regen) =

	0	1	2	3	4
0	"T1_prime (K)"	"T3 (K)"	"T4_prime (K)"	"v3 (m^3/kg)"	"v4_prime (m^3/kg)"
1	314.6944	80	42.2073	0.3331	0.8733

	5	6	7	8	9
	"Qa_by_m (kJ/kg)"	"W_by_m (kJ/kg)"	"COP"	"COP_i"	"FOM"
	74.9482	$1.004 \cdot 10^3$	0.0746	0.2453	0.3043

Qa_by_m(P2, T5, epsilon_regen) := GMcycle_Refrig_He(P1, P2, T2, T5, eta_co, eta_ad, epsilon_regen)$_{1,5}$

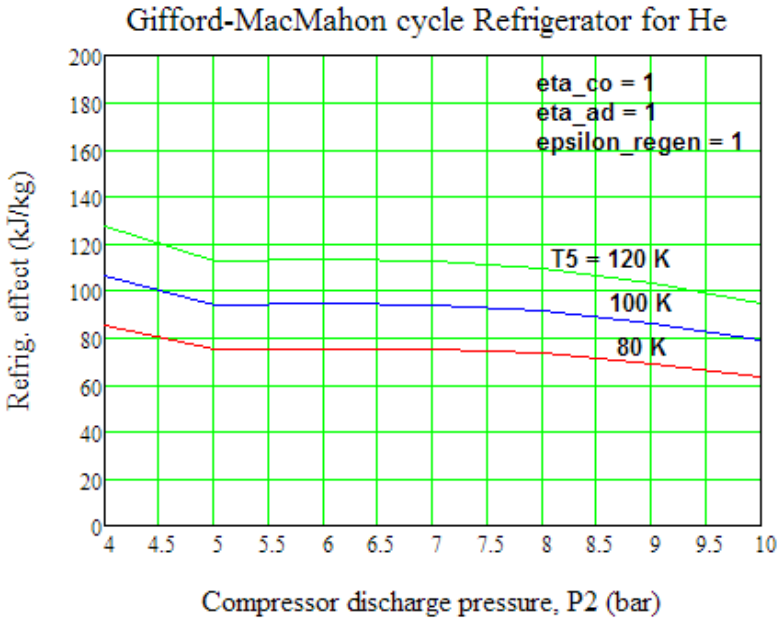
i.e. Qa_by_m(P2, T5, epsilon_regen) = 74.948 **kJ/kg ... Ans.**

COP(P2, T5, epsilon_regen) := GMcycle_Refrig_He(P1, P2, T2, T5, eta_co, eta_ad, epsilon_regen)$_{1,7}$

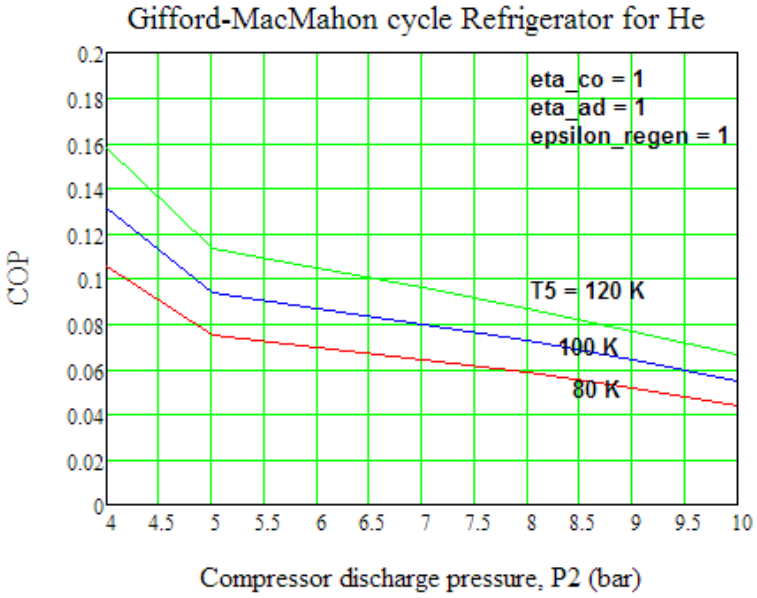
i.e. COP(P2, T5, epsilon_regen) = 0.075 **...Ans.**

Plot refrign. effect and COP for cold space temp T5= 80, 100 and 120 K, as compressor discharge pressure varies from 4 bar to 10 bar.

P2 := 4, 5.. 10 ...define a range variable



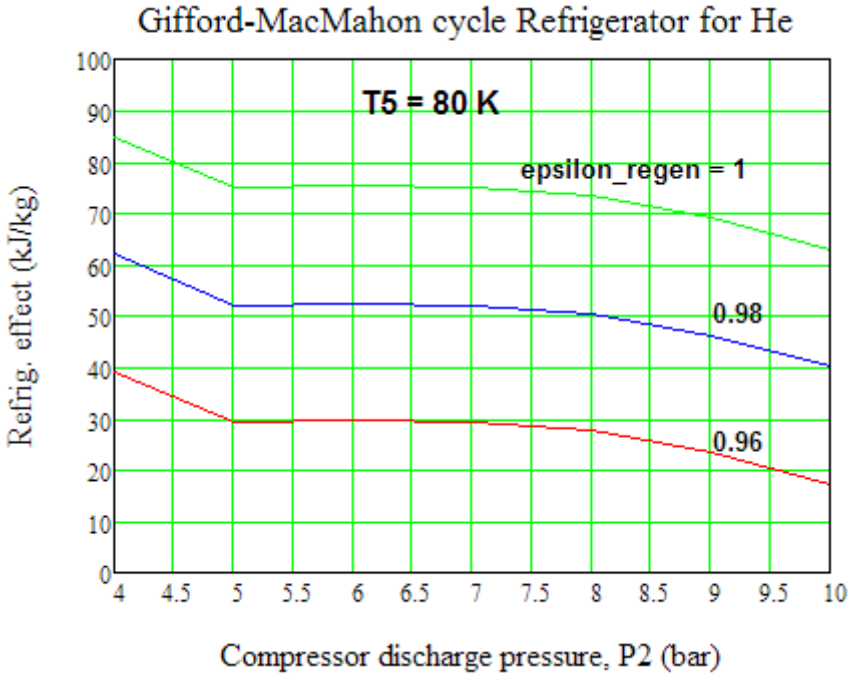
Download free eBooks at bookboon.com

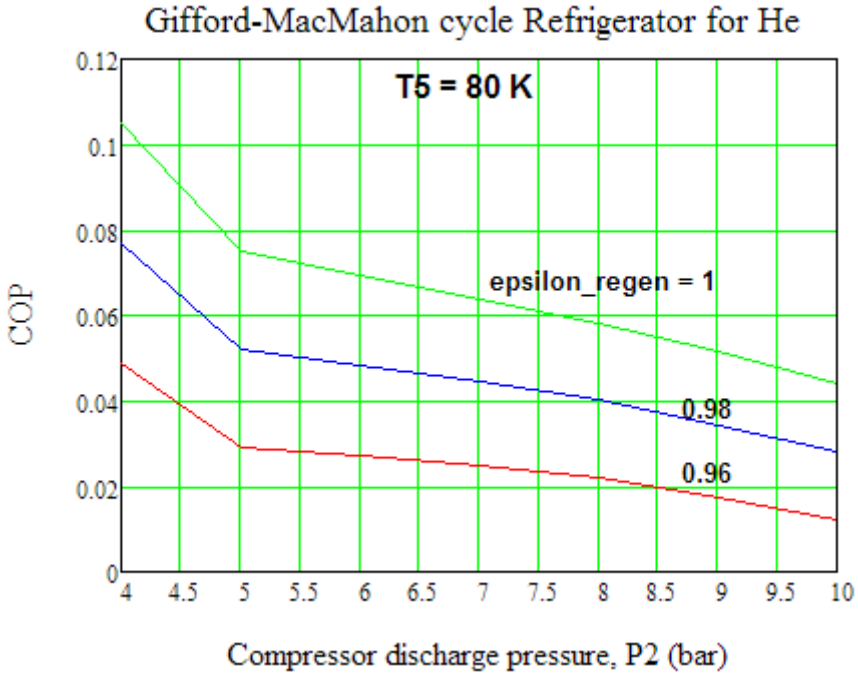


The Wake
the only emission we want to leave behind
Low-speed Engines Medium-speed Engines Turbochargers Propellers Propulsion Packages PrimeServ
The design of eco-friendly marine power and propulsion solutions is crucial for MAN Diesel & Turbo.
Power competencies are offered with the world's largest engine programme – having outputs spanning
from 450 to 87,220 kW per engine. Get up front!
Find out more at www.mandieselturbo.com
Engineering the Future – since 1758.
MAN Diesel & Turbo
MAN

Download free eBooks at bookboon.com

Click on the ad to read more





Prob. 4.2.31 Write a Mathcad Program to determine the effectiveness of a practical regenerator of a GM cycle refrigerator, consisting of copper wire screen matrix housed in a perspex displacer. Helium is the working gas.

Download free eBooks at bookboon.com

Frst, we need the porosity of the regenerator.

This is calculated knowing the physical dimensions of the regenerator and the mass of copper screens.

porosity = (Total vol. - metal vol.)/Total vol.

$$\text{porosity}(LR, DR, mcopper, \rho\_cu) = \frac{\frac{\pi \cdot DR^2}{4} \cdot LR - \frac{mcopper}{\rho\_cu}}{\frac{\pi \cdot DR^2}{4} \cdot LR}$$

where LR, DR ... are length and dia of regenerator housing, mcopper = mass of copper, ρ_cu = density of copper

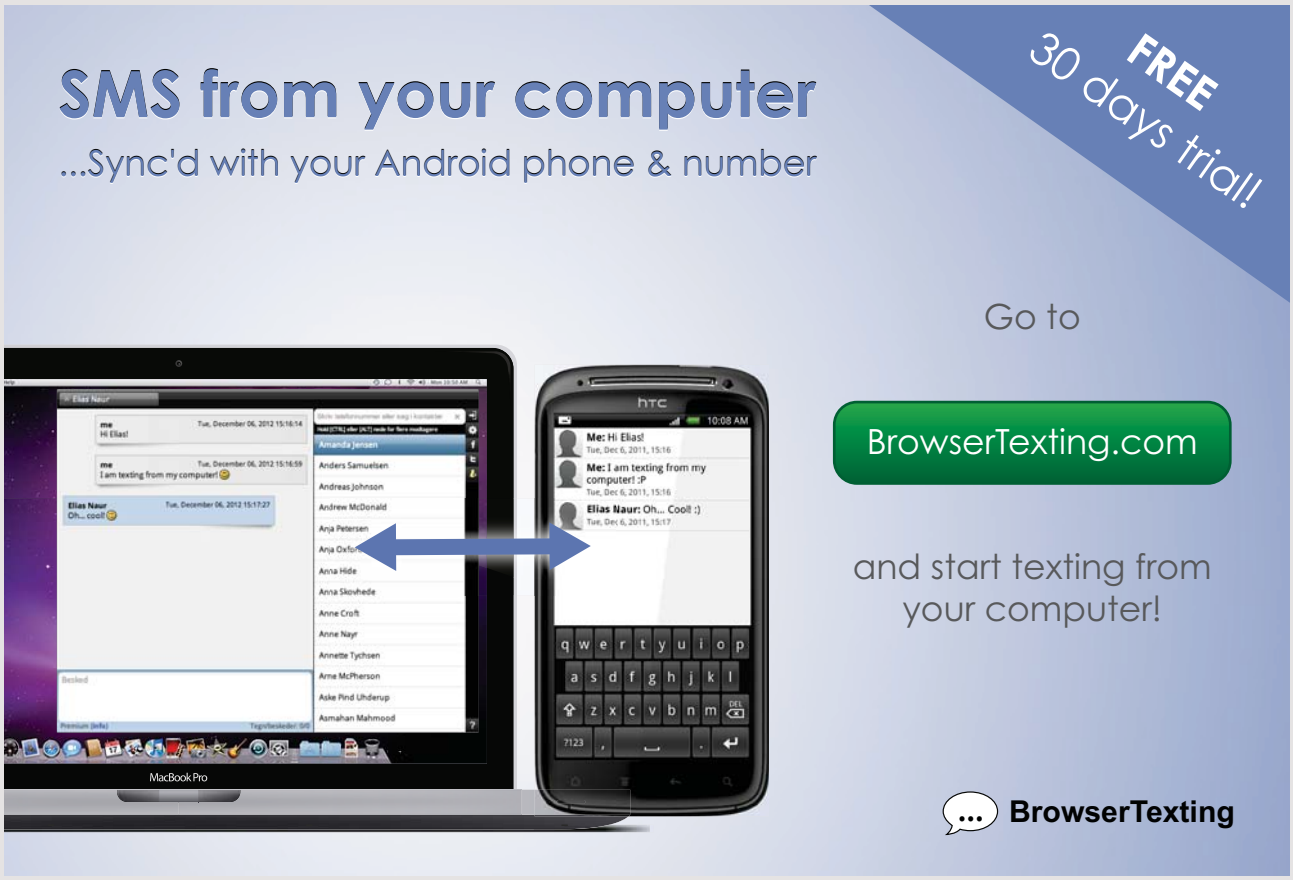
We have, for sp. heat of copper:

	T (K)		cp (J/g.K)
temp :=	20	cp :=	7.70E-03
	30		2.70E-02
	40		6.00E-02
	50		9.90E-02
	60		1.37E-01
	70		1.73E-01
	80		2.05E-01
	90		2.32E-01
	100		2.54E-01
	110		2.71E-01
	120		2.88E-01
	130		3.01E-01
	140		3.13E-01
	150		3.23E-01
	160		3.32E-01
	170		3.39E-01
	180		3.46E-01
	190		3.51E-01
	200		3.56E-01

Download free eBooks at bookboon.com

210	3.60E-01
220	3.64E-01
230	3.68E-01
240	3.71E-01
250	3.74E-01
260	3.76E-01
270	3.79E-01
280	3.81E-01
290	3.84E-01
300	3.86E-01
320	3.91E-01

$cpcopper(T) := linterp(temp, cp, T) \cdot 10^3$ $cpcopper(250) = 374$ J/kg.K



Download free eBooks at bookboon.com
Click on the ad to read more

Viscosity, thermal conductivity and sp. heat of helium gas [29]:

Upto 20 bar, visc. and th. cond. are independent of pr., depend only on temp (Ref: Arkharov):

$$khelium(T) := 0.1465 \cdot \left(\frac{T}{293}\right)^{1.5} \cdot (293 + 78) \cdot (T + 78)^{-1} \qquad khelium(250) = 0.131 \quad W/m.K$$

$$\mu helium(T) := 18.9 \cdot 10^{-6} \cdot \left(\frac{T}{293}\right)^{1.5} \cdot (293 + 78) \cdot (T + 78)^{-1} \qquad \mu helium(250) = 1.685 \times 10^{-5} \quad Pa.s$$

Properties of helium at 1.013 bar: Ref: NIST [28]

T(K) rho(kg/m^3) h (kJ/kg) s(kJ/kg.K) cp (kJ/kg.K)

$$V1ATM := \begin{pmatrix} 5 & 11.975 & 26.606 & 6.1823 & 6.7698 \\ 15 & 3.2719 & 82.176 & 12.394 & 5.2961 \\ 25 & 1.9476 & 134.72 & 15.079 & 5.2295 \\ 35 & 1.3897 & 186.91 & 16.835 & 5.2108 \\ 45 & 1.0809 & 238.97 & 18.144 & 5.2031 \\ 55 & 0.88458 & 290.98 & 19.188 & 5.1994 \\ 65 & 0.74867 & 342.96 & 20.056 & 5.1972 \\ 75 & 0.64899 & 394.93 & 20.8 & 5.196 \\ 85 & 0.57274 & 446.89 & 21.45 & 5.1951 \\ 95 & 0.51254 & 498.83 & 22.028 & 5.1946 \\ 105 & 0.46379 & 550.78 & 22.548 & 5.1942 \\ 115 & 0.42351 & 602.72 & 23.02 & 5.1939 \\ 125 & 0.38967 & 654.66 & 23.453 & 5.1937 \\ 135 & 0.36084 & 706.59 & 23.853 & 5.1935 \\ 145 & 0.33598 & 758.53 & 24.224 & 5.1934 \\ 155 & 0.31432 & 810.46 & 24.57 & 5.1933 \\ 165 & 0.29529 & 862.39 & 24.895 & 5.1933 \\ 175 & 0.27843 & 914.33 & 25.201 & 5.1932 \\ 185 & 0.2634 & 966.26 & 25.489 & 5.1932 \\ 195 & 0.2499 & 1018.2 & 25.763 & 5.1931 \\ 205 & 0.23772 & 1070.1 & 26.022 & 5.1931 \end{pmatrix}$$

Download free eBooks at bookboon.com

215	0.22667	1122.1	26.27	5.1931
225	0.2166	1174	26.506	5.1931
235	0.20739	1225.9	26.732	5.193
245	0.19893	1277.8	26.948	5.193
255	0.19114	1329.8	27.156	5.193
265	0.18393	1381.7	27.356	5.193
275	0.17724	1433.6	27.548	5.193
285	0.17103	1485.6	27.733	5.193
295	0.16523	1537.5	27.912	5.193
305	0.15982	1589.4	28.086	5.193
315	0.15475	1641.4	28.253	5.193
325	0.14999	1693.3	28.415	5.193
335	0.14551	1745.2	28.573	5.193
345	0.1413	1797.1	28.726	5.193
355	0.13732	1849.1	28.874	5.193
365	0.13356	1901	29.018	5.193

$T1ATM := V1ATM^{\langle 0 \rangle}$ +

$DENS1ATM := V1ATM^{\langle 1 \rangle}$ $H1ATM := V1ATM^{\langle 2 \rangle}$ $S1ATM := V1ATM^{\langle 3 \rangle}$ $CP1ATM := V1ATM^{\langle 4 \rangle}$

$rho1atm(T) := linterp(T1ATM, DENS1ATM, T)$ **..dens. at 1 atm., kg/m^3** $rho1atm(210) = 0.232$

$h1atm(T) := linterp(T1ATM, H1ATM, T)$ **..enth. at 1 atm., kJ/kg** $h1atm(210) = 1.096 \times 10^3$

$s1atm(T) := linterp(T1ATM, S1ATM, T)$ **..entropy at 1 atm., kJ/kg.K** $s1atm(210) = 26.146$

$cphe1atm(T) := linterp(T1ATM, CP1ATM, T) \cdot 10^3$ **..sp.heat at 1 atm., J/kg.K** $cphe1atm(210) = 5.193 \times 10^3$

Download free eBooks at bookboon.com

Now, write the Function for regenerator calculations:

$$\text{Regen\_cu\_GM\_He}(PH, PL, TH, TL, N, d_i, d_o, lcyl, ddisp, ldisp, LR, DR, dw, p) :=$$

$$R \leftarrow 2078.5$$

$$gama \leftarrow 1.67$$

$$\rho\_cu \leftarrow 8874$$

$$T\_m \leftarrow \frac{TH + TL}{2}$$

$$A_{ht} \leftarrow \frac{4\cdot(1-p)}{dw}\cdot\pi\cdot\frac{DR^2}{4}\cdot LR$$

$$d_{eq} \leftarrow 4\cdot LR\cdot p\cdot\pi\cdot\frac{DR^2}{4\cdot A_{ht}}$$

$$V\_stroke \leftarrow \frac{\pi\cdot d_i^2}{4}\cdot(lcyl - ldisp)$$

$$m \leftarrow \frac{PH\cdot 10^5\cdot V\_stroke}{R\cdot TL}\cdot\frac{N}{60}$$

Download free eBooks at bookboon.com
Click on the ad to read more

$$G \leftarrow \frac{m}{\left(\frac{p \cdot \pi \cdot DR^2}{4}\right)}$$

$$Re \leftarrow \frac{G \cdot d_{eq}}{\mu helium(T\_m)}$$

$$Pr \leftarrow \frac{cphe1atm(T\_m) \cdot \mu helium(T\_m)}{khelium(T\_m)}$$

$$St \leftarrow 0.68 \cdot Re^{-0.407} \cdot Pr^{\left(-\frac{2}{3}\right)}$$

$$h \leftarrow St \cdot G \cdot cphe1atm(T\_m)$$

$$NTU \leftarrow \frac{h \cdot A_{ht}}{2 \cdot cphe1atm(T\_m) \cdot m}$$

$$ms \leftarrow \frac{\pi \cdot DR^2}{4} \cdot LR \cdot (1 - p) \cdot \rho\_cu$$

$$tau \leftarrow \frac{1}{\left(\frac{N}{60}\right)} \cdot \frac{1}{2}$$

$$CR \leftarrow \frac{ms \cdot cpcopper(T\_m)}{cphe1atm(T\_m) \cdot m \cdot tau}$$

$$\Delta T \leftarrow \frac{m \cdot (TH - TL) \cdot tau \cdot cphe1atm(T\_m)}{ms \cdot cpcopper(T\_m)}$$

$$I \leftarrow \frac{1}{1 + NTU}$$

$$Qloss\_reg \leftarrow \frac{I \cdot m \cdot (TH - TL) \cdot cphe1atm(T\_m)}{2}$$

$$f \leftarrow \frac{860 \cdot (1 - p)}{Re} + \frac{2.2 \cdot p}{Re^{0.1}}$$

$$\rho gas \leftarrow \frac{PH \cdot 10^5}{R \cdot T\_m}$$

$$Q\_ideal \leftarrow (PH - PL) \cdot 10^5 \cdot V\_stroke \cdot \frac{N}{60}$$

$$\Delta P \leftarrow \frac{f \cdot LR}{d_{eq}} \cdot \frac{G^2}{2 \cdot \rho gas}$$

"Qideal(W)"	"m_He(kg/s)"	"A_ht(m^2)"	"Re"	"h(W/m^2.K)"	"NTU"	"Regen. Ineffcy."	"f"	"DELTAP(Pa)"	"Qloss_regen(W)"
Q_ideal	m	A_{ht}	Re	h	NTU	I	f	ΔP	Qloss_reg

Download free eBooks at bookboon.com

In the above Function we have:

Inputs:

PH,PL … high and low pressures, bar; TH, TL .. high and low temp, K

N … rpm; di, do … ID and OD of cylinder, m

lcyl … length of stainless steel cylinder, m

ddisp, ldisp … dia and length of Perspex displacer, m

LR, DR … length and dia of regenerator housing, m

dw … dia of regenerator material, i.e. copper wire mesh

p … porosity of regenerator filling

Outputs:

Outputs are presented in a matrix, containing:

Qideal … Ideal refrigeration produced (= (PH – PL) □ Vstroke), W

m_He … mass flow rate of helium, kg/s; A_ht … heat transfer area, m^2

Re … Reynolds number; h … heat transfer coeff., W/m^2.K

NTU … No. of Transfer Units; I … Regen. Inefficiency (= 1 – efficiency)

f … friction factor; ΔP … pressure drop in regenerator, Pa

Qloss_reg … heat loss in regenerator, W

Download free eBooks at bookboon.com

Prob. 4.2.32: In a practical GM cycle refrigerator using Helium, following are the data. Apply the Function written above and determine the various parameters of interest.

PH := 9 bar,... high pr. in refrigerator PL := 1 bar, ..low pr. N := 130 rpm

$R := \frac{8314}{4.003}$ J/kg.K i.e. $R = 2.0769 \times 10^3$ J/kg.K

TH := 310 K TL := 80 K gama := 1.67 ...ratio of sp.heats

Data for cyl. and displacer:

1. SS cylinder: ID = 48.6 mm, OD = 50 mm, length = 240 mm

$d_i := 0.0486$ m $d_o := 0.05$ m lcyl := 0.24 m

2. Perspex displacer: dia = 47.3 mm, length = 220 mm with a housing for regenerator of dia 42 mm and 101 mm deep; 2 holes of 2 mm dia for gas passage

ddisp := 0.0473 m LR := 0.101m dhole := 0.002 m

ldisp := 0.22 m DR := 0.042 m nhole := 2

Brain power
By 2020, wind could provide one-tenth of our planet's electricity needs. Already today, SKF's innovative know-how is crucial to running a large proportion of the world's wind turbines.
Up to 25 % of the generating costs relate to maintenance. These can be reduced dramatically thanks to our systems for on-line condition monitoring and automatic lubrication. We help make it more economical to create cleaner, cheaper energy out of thin air.
By sharing our experience, expertise, and creativity, industries can boost performance beyond expectations.
Therefore we need the best employees who can meet this challenge!
The Power of Knowledge Engineering
Plug into The Power of Knowledge Engineering.
Visit us at www.skf.com/knowledge
SKF
Download free eBooks at bookboon.com
Click on the ad to read more

So, we have:

$DR := 0.042$ m,...Regen. housing dia.

$LR := 0.101$ m,...Regen. housing length

$dw := 8 \cdot 10^{-5}$ m....wire dia for 150 mesh wire screen

$\rho\_cu := 8874$ kg/m^3,...density of matrix material (copper)

$m\_cu := 0.312$ kg...mass of regen. material, by weighing

Then, Porosity:

$$p := \frac{\frac{\pi \cdot DR^2}{4} \cdot LR - \frac{m\_cu}{\rho\_cu}}{\frac{\pi \cdot DR^2}{4} \cdot LR} \qquad \text{i.e.} \qquad p = 0.749$$

Using the Function written above, we get:

$$Regen\_cu\_GM\_He(PH, PL, TH, TL, N, d_i, d_o, lcyl, ddisp, ldisp, LR, DR, dw, p) =$$

	0	1	2	3	4
0	"Qideal(W)"	"m_He(kg/s)"	"A_ht(m^2)"	"Re"	"h(W/m^2.K)"
1	64.309	$4.351 \cdot 10^{-4}$	1.758	7.17	867.691

5		6	7	8	9
	"NTU"	"Regen. Ineffcy."	"f"	"DELTAP(Pa)"	"Qloss_regen(W)"
	337.541	$2.954 \cdot 10^{-3}$	31.489	528.479	0.768

We write various parameters as functions of PH, PL and N, for easy plotting purpose: And, we get:

$$Qideal(PH, PL, N) := Regen\_cu\_GM\_He(PH, PL, TH, TL, N, d_i, d_o, lcyl, ddisp, ldisp, LR, DR, dw, p)_{1,0}$$

$$A\_ht(PH, PL, N) := Regen\_cu\_GM\_He(PH, PL, TH, TL, N, d_i, d_o, lcyl, ddisp, ldisp, LR, DR, dw, p)_{1,2}$$

$$Re(PH, PL, N) := Regen\_cu\_GM\_He(PH, PL, TH, TL, N, d_i, d_o, lcyl, ddisp, ldisp, LR, DR, dw, p)_{1,3}$$

Download free eBooks at bookboon.com

$h(PH,PL,N) := Regen\_cu\_GM\_He(PH,PL,TH,TL,N,d_i,d_o,lcyl,ddisp,ldisp,LR,DR,dw,p)_{1,4}$

$NTU(PH,PL,N) := Regen\_cu\_GM\_He(PH,PL,TH,TL,N,d_i,d_o,lcyl,ddisp,ldisp,LR,DR,dw,p)_{1,5}$

$Ineffcy(PH,PL,N) := Regen\_cu\_GM\_He(PH,PL,TH,TL,N,d_i,d_o,lcyl,ddisp,ldisp,LR,DR,dw,p)_{1,6}$

$DELTAP(PH,PL,N) := Regen\_cu\_GM\_He(PH,PL,TH,TL,N,d_i,d_o,lcyl,ddisp,ldisp,LR,DR,dw,p)_{1,8}$

$Qloss\_regen(PH,PL,N) := Regen\_cu\_GM\_He(PH,PL,TH,TL,N,d_i,d_o,lcyl,ddisp,ldisp,LR,DR,dw,p)_{1,9}$

And, results:

$Qideal(PH,PL,N) = 64.309$ W

$A\_ht(PH,PL,N) = 1.758$ m^2

$Re(PH,PL,N) = 7.17$

$h(PH,PL,N) = 867.691$ W/m^2.K

$NTU(PH,PL,N) = 337.541$

$Ineffcy(PH,PL,N) = 2.954 \times 10^{-3}$

$DELTAP(PH,PL,N) = 528.479$ Pa

$Qloss\_regen(PH,PL,N) = 0.768$ W

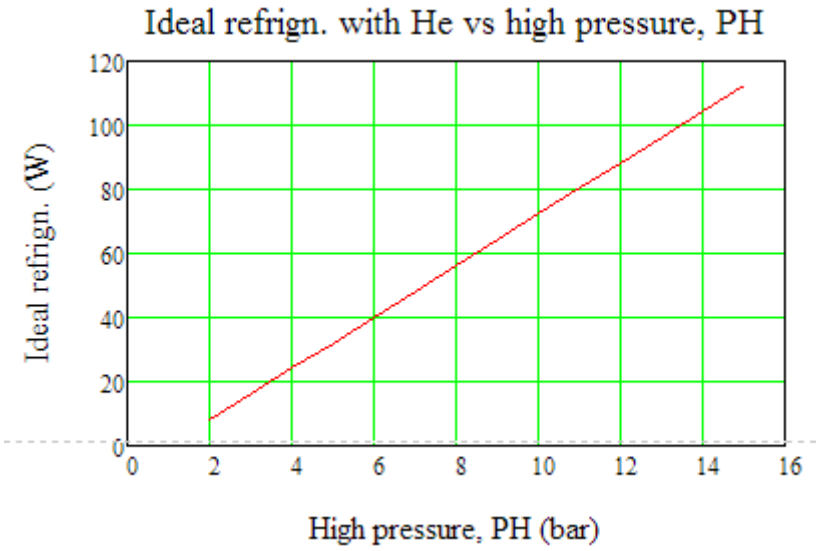
Download free eBooks at bookboon.com

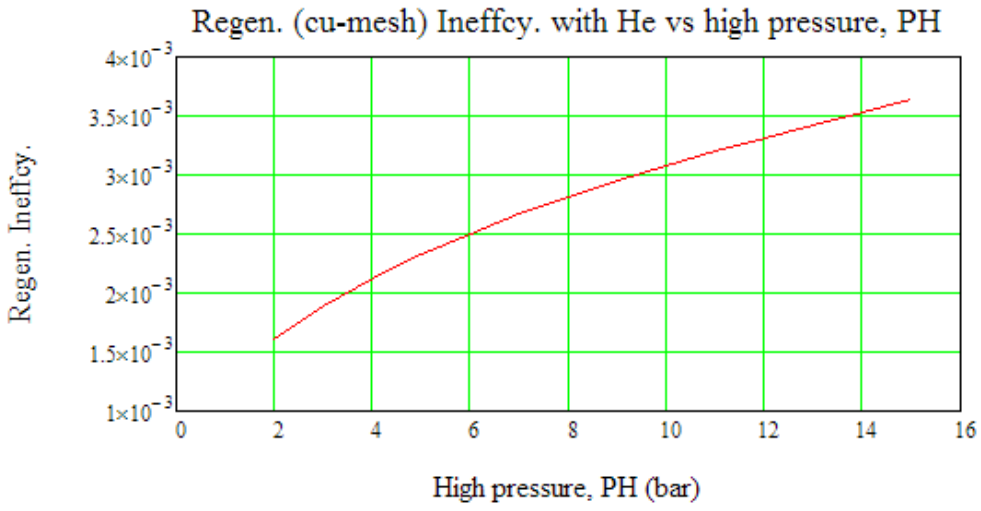
Effect of changing PH:

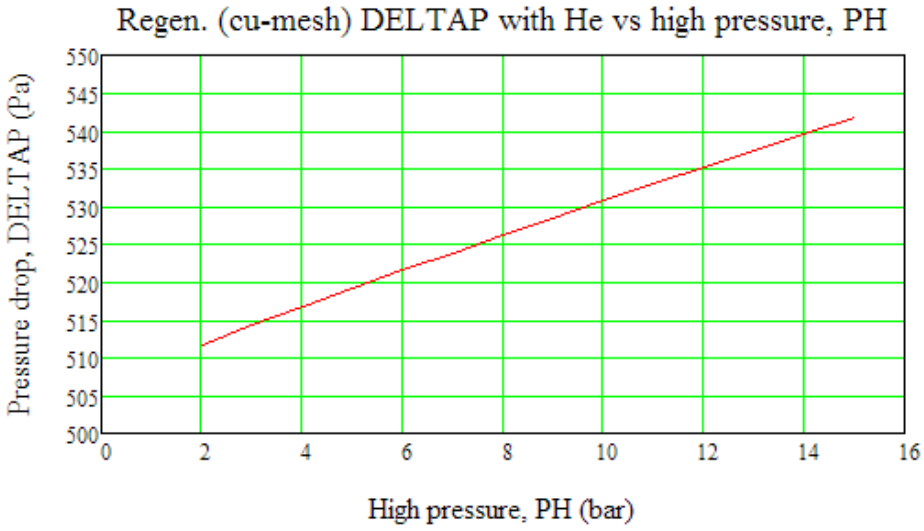
PH := 2,3.. 15 bar ... define a range variable

PH =	Ineffcy(PH,PL,N)	DELTAP(PH,PL,N)	Qloss_regen(PH,PL,N)	Qideal(PH,PL,N)
2	$1.604 \cdot 10^{-3}$	511.641	0.093	8.039
3	$1.891 \cdot 10^{-3}$	514.223	0.164	16.077
4	$2.125 \cdot 10^{-3}$	516.719	0.245	24.116
5	$2.327 \cdot 10^{-3}$	519.153	0.336	32.155
6	$2.506 \cdot 10^{-3}$	521.538	0.434	40.193
7	$2.667 \cdot 10^{-3}$	523.884	0.539	48.232
8	$2.816 \cdot 10^{-3}$	526.196	0.65	56.271
9	$2.954 \cdot 10^{-3}$	528.479	0.768	64.309
10	$3.083 \cdot 10^{-3}$	530.738	0.89	72.348
11	$3.204 \cdot 10^{-3}$	532.973	1.018	80.387
12	$3.32 \cdot 10^{-3}$	535.188	1.15	88.425
13	$3.429 \cdot 10^{-3}$	537.385	1.287	96.464
14	$3.534 \cdot 10^{-3}$	539.566	1.428	104.503
15	$3.634 \cdot 10^{-3}$	541.73	1.574	112.541

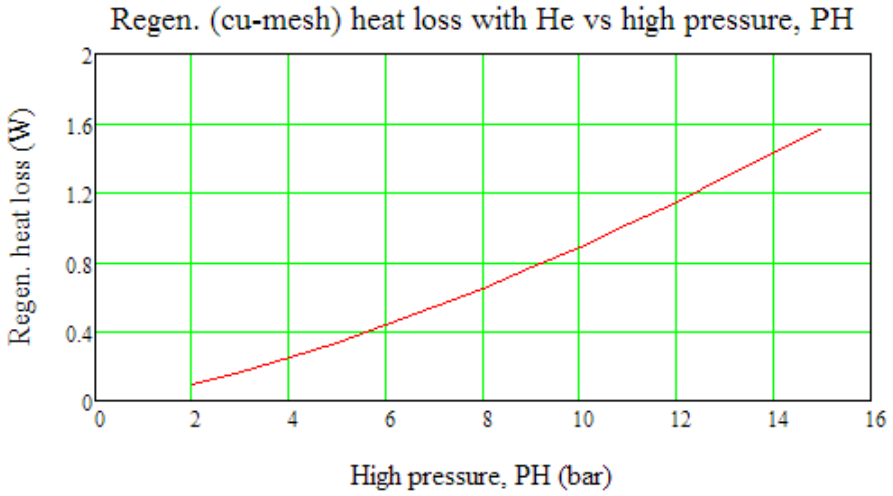
Download free eBooks at bookboon.com
Click on the ad to read more



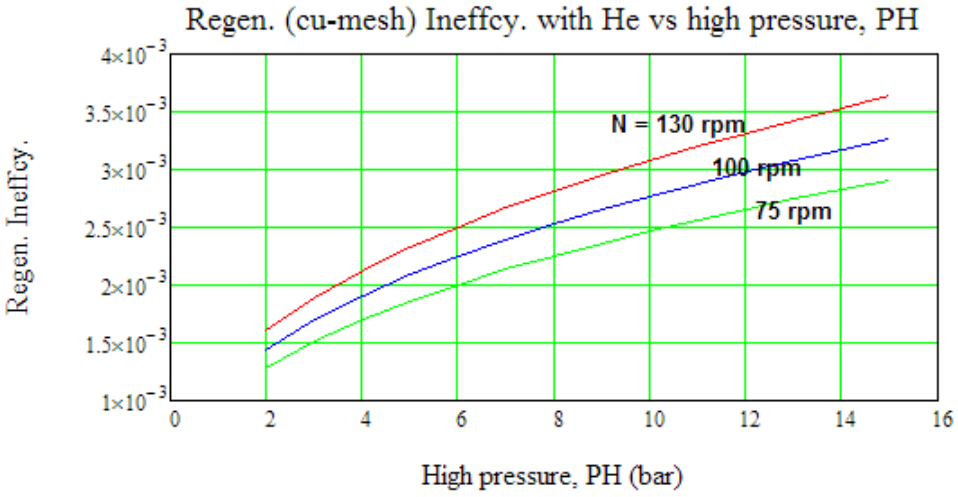




Download free eBooks at bookboon.com



Variation of regen. ineffcy. with speed, N:



==========

Prob. 4.2.33: Write a Mathcad Function to find out the effectiveness of a regenerator by Λ-π method.

Barron, Ref. [1], gives a Table of efficiency values for different Lambda and PI values, for **'symmetric and balanced regenerators'**, i.e.

Reduced length: $\Lambda_h = \frac{h_h \cdot A_h}{mdot_h \cdot c_h}$ and, $\Lambda_c = \frac{h_c \cdot A_c}{mdot_c \cdot c_c}$

Reduced period: $\Pi_h = \frac{h_h \cdot A_h \cdot P_h}{m_{sh} \cdot c_s}$ and, $\Pi_c = \frac{h_c \cdot A_c \cdot P_c}{m_{sc} \cdot c_s}$

Download free eBooks at bookboon.com

Regenerator is said to be *symmetric*, if:

$$\Pi_h = \Pi_c$$

And, regenerator is *balanced*, if:

$$\Lambda_h / \Pi_h = \Lambda_c / \Pi_c$$

LIGS University
based in Hawaii, USA
is currently enrolling in the Interactive Online BBA, MBA, MSc, DBA and PhD programs:
- enroll by October 31st, 2014 and
- save up to 11% on the tuition!
- pay in 10 installments / 2 years
- Interactive Online education
- visit www.ligsuniversity.com to find out more!

Note: LIGS University is not accredited by any nationally recognized accrediting agency listed by the US Secretary of Education.
More info here.

Download free eBooks at bookboon.com
Click on the ad to read more

We use that Table to write the desired Function:

First, the data is copied into a Mathcad matrix:

Efffcy_MM :=

Lambda	PI = 0	5	10	15	20	25	30	35	40	45	50
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.7143	0.6390	0.4690	0.3260	0.2500	0.202	0.1670	0.1430	0.1250	0.1120	0.1000
10	0.8333	0.8090	0.7380	0.6100	0.4940	0.4020	0.3330	0.2840	0.2500	0.2220	0.2000
15	0.8824	0.8660	0.8400	0.7840	0.6930	0.5850	0.4980	0.4300	0.3750	0.3320	0.3000
20	0.9091	0.9000	0.8860	0.8560	0.8110	0.7560	0.6510	0.5780	0.5000	0.4430	0.4000
25	0.9259	0.9200	0.9110	0.8950	0.8710	0.8270	0.7700	0.6980	0.6200	0.5550	0.5000
30	0.9375	0.9330	0.9270	0.9180	0.9030	0.8810	0.8450	0.7970	0.7270	0.6540	0.5980
35	0.9459	0.9430	0.9390	0.9320	0.9220	0.9070	0.8880	0.8560	0.8100	0.7540	0.6920
40	0.9524	0.9500	0.9470	0.9420	0.9350	0.9250	0.9120	0.8910	0.8650	0.8200	0.7730
45	0.9574	0.9550	0.9530	0.9490	0.9440	0.9380	0.9280	0.9140	0.8980	0.8710	0.8350
50	0.9615	0.9610	0.9580	0.9550	0.9510	0.9460	0.9390	0.9290	0.9150	0.8970	0.8750
100	0.9804	0.9800	0.9790	0.9790	0.9780	0.9770	0.9760	0.9750	0.9730	0.9720	0.9700
200	0.9901	0.9900	0.9890	0.9890	0.9890	0.9890	0.9890	0.9880	0.9880	0.9880	0.9880

In the above matrix, first column gives Lambda values; next columns give effcy values, respectively, for values of PI = 0, 5, 10, … 50.

Now, extract the different columns in to different vectors:

LAMBDA := Efffcy_MM$^{\langle 0 \rangle}$

EFFCY_M$_0$:= Efffcy_MM$^{\langle 1 \rangle}$ EFFCY_M$_5$:= Efffcy_MM$^{\langle 2 \rangle}$ EFFCY_M$_{10}$:= Efffcy_MM$^{\langle 3 \rangle}$

EFFCY_M$_{15}$:= Efffcy_MM$^{\langle 4 \rangle}$ EFFCY_M$_{20}$:= Efffcy_MM$^{\langle 5 \rangle}$ EFFCY_M$_{25}$:= Efffcy_MM$^{\langle 6 \rangle}$

EFFCY_M$_{30}$:= Efffcy_MM$^{\langle 7 \rangle}$ EFFCY_M$_{35}$:= Efffcy_MM$^{\langle 8 \rangle}$ EFFCY_M$_{40}$:= Efffcy_MM$^{\langle 9 \rangle}$

EFFCY_M$_{45}$:= Efffcy_MM$^{\langle 10 \rangle}$ EFFCY_M$_{50}$:= Efffcy_MM$^{\langle 11 \rangle}$

Download free eBooks at bookboon.com

And, for any value of Lambda for a given value of PI, efficiency is found by linear interpolation:

With this background, now we write the desired Function for efficiency of regenerator:

First, have a vector of various PI values for which we have efficiency values in the Table:

PI_values := (0 5 10 15 20 25 30 35 40 45 50) ..values of PI

Now, the Function:

```
Effcy_Regen(Lambda, PI) := return "Lambda must be between 0 and 200" if Lambda < 0 ∨ Lambda > 200
                           return "PI should be between 0 and 50" if PI < 0 ∨ PI > 50
                           for k ∈ 0.. 10
                              if PI = PI_values[0,k]
                                 effcy ← linterp[(LAMBDA), (EFFCY_M[PI_values[0,k]]), Lambda]
                                 return ("Lambda" "PI" "effcy")
                                        (Lambda   PI   effcy)
                              if PI > (PI_values[0,k]) ∧ PI < PI_values[0,k+1]
                                 PL ← PI_values[0,k]
                                 PH ← PI_values[0,k+1]
                                 A ← linterp[(LAMBDA), (EFFCY_M[PL]), Lambda]
                                 B ← linterp[(LAMBDA), (EFFCY_M[PH]), Lambda]
                                 effcy ← A + ((PI − PL)/(PH − PL))·(B − A)
                                 return ("Lambda" "PI" "effcy")
                                        (Lambda   PI   effcy)
```

Ex: Lambda := 12 PI := 30

$$\text{Effcy\_Regen}(\text{Lambda}, \text{PI}) = \begin{pmatrix} \text{"Lambda"} & \text{"PI"} & \text{"effcy"} \\ 12 & 30 & 0.399 \end{pmatrix}$$

Download free eBooks at bookboon.com

Prob. 4.2.34: Work out a problem to illustrate the use of the above Function:

A regenerator is constructed of 1.5 mm dia metal shots (density = 7800 kg/m^3, sp. heat = 0.84 kJ/kg.K), enclosed in a tube having an inside dia of 40 mm. Porosity of matrix is 0.4, and the heating and cooling periods are equal, at 2 seconds. The fluid flowing through the regenerator is gaseous helium at an average temp of 190 K and average pressure of 405.3 kPa. Mass flow rate of helium during heating and cooling periods is equal to 35 g/s. Determine the required length of regenerator, if an effectiveness of 95% is desired. [1]

Data:

Let LR be the length of regenerator.

DR := 0.04 m .. regen. dia

e_v := 0.4 ...porosity

Ds := 0.0015 m ... dia of solid material, i.e. metal shots

ρs := 7800 kg/m^3 ... density of solid, i.e. metal shots

cs := 840 J/kg.K .... sp. heat of solid

TURN TO THE EXPERTS FOR
SUBSCRIPTION CONSULTANCY

Subscrybe is one of the leading companies in Europe when it comes to innovation and business development within subscription businesses.

We innovate new subscription business models or improve existing ones. We do business reviews of existing subscription businesses and we develope acquisition and retention strategies.

Learn more at linkedin.com/company/subscrybe or contact
Managing Director Morten Suhr Hansen at mha@subscrybe.dk

SUBSCRYBE - to the future

Download free eBooks at bookboon.com
Click on the ad to read more

$mdot := 0.035$ kg/s ... mass flow rate of helium gas

$P := 2$ s ... period

Calculations:

Frontal area:

$$A_{fr} := \frac{\pi \cdot DR^2}{4} \quad \text{i.e.} \quad A_{fr} = 1.257 \times 10^{-3} \quad \text{m^2}$$

Therefore, free flow area:

$$A_{ff} := e_v \cdot A_{fr} \quad \text{i.e.} \quad A_{ff} = 5.027 \times 10^{-4} \quad \text{m^2}$$

Also, volume per unit length:

$$V_0 := A_{fr} \quad \text{i.e.} \quad V_0 = 1.257 \times 10^{-3} \quad \text{m^3}$$

Then, mass velocity, G:

$$G := \frac{mdot}{A_{ff}} \quad \text{i.e.} \quad G = 69.63 \quad \text{kg/s.m^2}$$

Properties of helium at 190 K and 405.3 kPa:

$$R_{He} := \frac{8314}{4.003} \quad \text{i.e.} \quad R_{He} = 2.077 \times 10^3 \quad \text{J/kg.K}$$

$P1 := 405.3 \cdot 10^3$ Pa $T := 190$ K

density of helium gas: $\rho helium(P1, T) := \frac{P1}{R_{He} \cdot T}$

i.e. $\rho helium(P1, T) = 1.027$ kg/m^3

Upto 20 bar, thermal cond. and viscosity of gases are independent of pressure, depend only on temp. [29]. For Helium, we have:

Thermal cond. k:

$$khelium(T) := 0.1465 \cdot \left(\frac{T}{293}\right)^{1.5} \cdot (293 + 78) \cdot (T + 78)^{-1}$$

i.e. $khelium(190) = 0.106$ W/m.K

Download free eBooks at bookboon.com

Dyn. viscosity, μ:

$$\mu helium(T) := 18.9 \cdot 10^{-6} \cdot \left(\frac{T}{293}\right)^{1.5} \cdot (293 + 78) \cdot (T + 78)^{-1}$$

i.e. $\mu helium(190) = 1.366 \times 10^{-5}$ Pa.s

and, sp. heat, cp is given by:

$$cphelium(T) := \frac{khelium(T)}{\mu helium(T) \cdot \left(1.77 - \frac{0.45}{1.67}\right)}$$

i.e. $cphelium(190) = 5.166 \times 10^{3}$ J/kg.K

Prandtl No., Pr: $Pr(T) := \frac{cphelium(T) \cdot \mu helium(T)}{khelium(T)}$

i.e. $Pr(190) = 0.666$

Therefore:

Reynolds No.: $Re(T) := \frac{Ds \cdot G}{\mu helium(T)}$

i.e. $Re(T) = 7.645 \times 10^{3}$ **...Reynolds No.**

Colburn j-factor:

$$jH(T) := 0.255 \cdot \left(e_v \cdot Re(T)\right)^{\frac{-1}{3}}$$ i.e. $jH(T) = 0.018$

But, we have: $jH = \frac{h_c \cdot Pr^{\frac{2}{3}}}{G \cdot cphelium}$

Download free eBooks at bookboon.com

Therefore, heat transfer coeff., hc

$$h_c(T) := \frac{jH(T)\cdot G\cdot cphelium(T)}{Pr(T)^{\frac{2}{3}}}$$

i.e. $h_c(T) = 8.282 \times 10^3$ **W/m^2.K ..... heat transfer coeff.**

Heat transfer surface area (A) per unit length:

$$A := 6\cdot(1 - e_v)\cdot\frac{V_0}{Ds}$$ m^2/m length

i.e. $A = 3.016$ m^2/m length

Mass of matrix per unit length:

$$m_s := \rho s\cdot(1 - e_v)\cdot V_0$$

i.e. $m_s = 5.881$ kg/m length

"I studied English for 16 years but... ...I finally learned to speak it in just six lessons"
Jane, Chinese architect
ENGLISH OUT THERE
Click to hear me talking before and after my unique course download

Download free eBooks at bookboon.com

Click on the ad to read more

Now, since the regenerator is symmetric and balanced, apply the Λ-Π method:

$$\Pi(T) := \frac{h_c(T) \cdot A \cdot P}{m_s \cdot cs}$$

i.e. $\Pi(T) = 10.113$ **...dimensionless period**

Now, use the above written Mathcad Function to get Λ for a regenerator effectiveness of 0.95:

We have:

$PI := 10.113$ $effcy := 0.95$

$Lambda := 20$ ..trial value

Given

$Effcy\_Regen(Lambda, PI)_{1,2} = effcy$

$Lambda := Find(Lambda)$

i.e. $Lambda = 42.584$

But: $Lambda = \frac{h_c \cdot As}{mdot \cdot cp}$

Therefore, required surface area, As:

$$As = \frac{Lambda \cdot mdot \cdot cp}{h_c}$$

$$As(T) := \frac{Lambda \cdot mdot \cdot cphelium(T)}{h_c(T)}$$

i.e. $As(T) = 0.93$ **m^2 ... required total surface area of regenerator**

Therefore, required length of regenerator, LR:

$$LR(T) := \frac{As(T)}{A}$$

i.e. $LR(T) = 0.308$ **m... required length of regenerator .. Ans.**

Download free eBooks at bookboon.com

Check: dwell time of gas in regenerator:

$$tau\_d = LR \cdot A_{ff} \cdot \frac{\rho helium}{mdot} = \frac{0.308 \cdot 5.027 \cdot 10^{-4} \cdot 1.027}{0.035} = 0.004543 \quad s$$

…this dwell time is much less than the period of 2 seconds…as required to use the Function for regenerator effcy, written above.

Prob. 4.2.35: A regenerator is constructed of a matrix material with sp. heat = 1 kJ/kg.K, and a mass of 5.5 kg. The fluid flowing through the regenerator is a gas with sp. heat of 1.005 kJ/kg.K and a mass flow rate of 32 g/s for both heating and cooling periods. The heat transfer coeff for both the cooling and heating periods is 402 W/m^2.K and the surface area is 2.00 m^2. The heating and cooling periods are 2.5 minutes, each. Find the effectiveness of regenerator using the Lambda-PI method. [1]

Data:

ms := 5.5 kg ... mass of solid,

cs := 1000 J/kg.K .... sp. heat of solid

cpgas := 1005 J/kg.K

mdot := 0.032 kg/s ... mass flow rate of helium gas

P := 2.5·60 s ... period

h_c := 402 W/m^2.K

As := 2.0 m^2 ... surface area

Calculations:

$$Lambda := \frac{h_c \cdot As}{mdot \cdot cpgas} \qquad \text{i.e.} \quad Lambda = 25 \quad \text{....dimensionless length}$$

$$PI := \frac{h_c \cdot As \cdot P}{ms \cdot cs} \qquad \text{i.e.} \quad PI = 21.927 \quad \text{....dimensionless period}$$

Download free eBooks at bookboon.com

Therefore, effectiveness is found out by using the Mathcad Function written earlier:

$$\text{Effcy\_Regen}(\text{Lambda}, \text{PI}) = \begin{pmatrix} \text{"Lambda"} & \text{"PI"} & \text{"effcy"} \\ 25 & 21.927 & 0.854 \end{pmatrix}$$

i.e. Regenerator efficiency = 0.854 = 85.4%... Ans.

==========

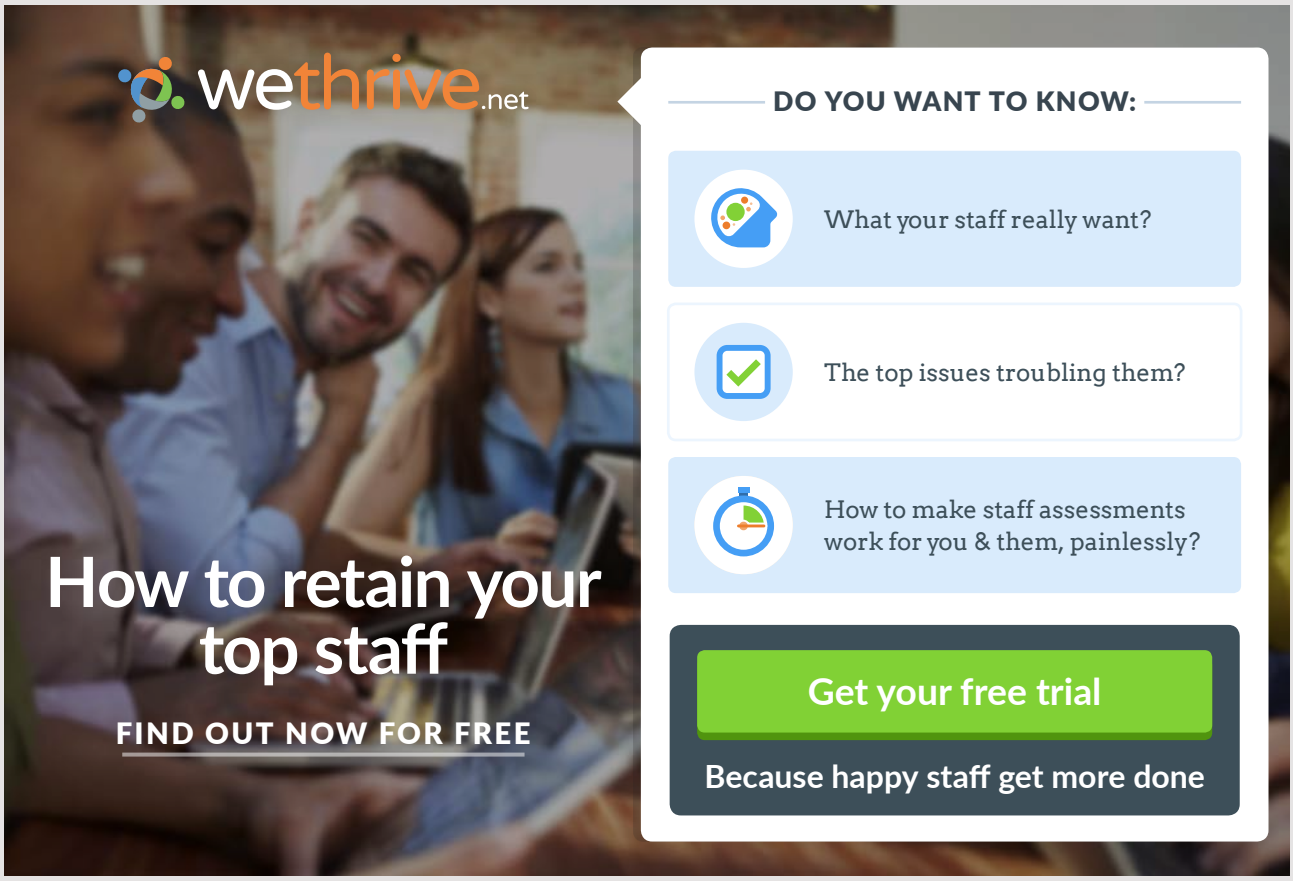
Prob.4.2.36 Write a Mathcad Function to calculate the effectiveness of a regenerator by NTU-ε method, when CR = (Cmin/Cmax) = 1.

We have:

$$NTU_0 := \frac{1}{C_{min}} \cdot \left(\frac{1}{hh \cdot Ah} + \frac{1}{hc \cdot Ac} \right)^{-1}$$

$$C_R := \frac{C_{min}}{C_{max}}$$

$$C_m := \frac{ms \cdot cs}{P0 \cdot C_{min}}$$


wethrive.net
How to retain your top staff
FIND OUT NOW FOR FREE
DO YOU WANT TO KNOW:
What your staff really want?
The top issues troubling them?
How to make staff assessments work for you & them, painlessly?
Get your free trial
Because happy staff get more done

Download free eBooks at bookboon.com
Click on the ad to read more

$P_0 = P_h + P_c$

$f = 1/ P_0$

$\varepsilon = Q\_dot /[C_{min}. (T_{h1} - T_{c1})]$

When CR is not equal to 1:

First, find the 'effective values' of NTU and Cm:

$$NTU_e := \frac{2 \cdot NTU_0 \cdot C_R}{(1 + C_R)}$$

$$C_{me} := \frac{2 \cdot C_m \cdot C_R}{(1 + C_R)}$$

Using these effective values, find the effectiveness (ε1) for a 'balanced' regenerator from Table.

Then, determine following parameter 'X':

$$X := \frac{1 + C_R}{2 \cdot C_R} \cdot \frac{\varepsilon_1}{1 - \varepsilon_1} \cdot (1 - C_R)$$

Finally, regenerator effectiveness is calculated as:

$$\varepsilon := \frac{1 - \exp(-X)}{1 - C_R \cdot \exp(-X)}$$

Download free eBooks at bookboon.com

Now, copy the Table 5.2 in Barron (Ref.[1]), in to a Mathcad Matrix:

Regen_M :=

NTU_0	Values of Cm: 0.8	1	1.25	1.5	2	3	5	10000000
0	0	0	0	0	0	0	0	0
0.5	0.315	0.322	0.326	0.328	0.33	0.332	0.333	0.3333
1	0.449	0.467	0.478	0.485	0.491	0.496	0.499	0.5
1.5	0.521	0.548	0.566	0.576	0.586	0.594	0.598	0.6
2	0.566	0.601	0.623	0.636	0.649	0.659	0.664	0.6667
2.5	0.599	0.639	0.664	0.679	0.694	0.705	0.711	0.7143
3	0.622	0.667	0.696	0.712	0.728	0.74	0.746	0.75
3.5	0.642	0.69	0.721	0.738	0.755	0.767	0.774	0.7778
4	0.659	0.709	0.741	0.759	0.776	0.789	0.796	0.8
4.5	0.673	0.724	0.758	0.776	0.794	0.807	0.814	0.8182
5	0.685	0.738	0.772	0.791	0.809	0.822	0.829	0.8333
5.5	0.696	0.749	0.785	0.803	0.821	0.834	0.842	0.8462
6	0.705	0.759	0.796	0.814	0.832	0.845	0.853	0.8571
6.5	0.713	0.768	0.805	0.824	0.842	0.855	0.862	0.8667
7	0.721	0.776	0.814	0.833	0.85	0.863	0.87	0.875
7.5	0.728	0.784	0.822	0.84	0.858	0.871	0.878	0.8824
8	0.734	0.79	0.829	0.847	0.865	0.877	0.884	0.8889
8.5	0.740	0.796	0.835	0.854	0.871	0.883	0.89	0.8947
9	0.746	0.802	0.841	0.859	0.876	0.888	0.895	0.9
9.5	0.751	0.807	0.846	0.864	0.881	0.893	0.9	0.9048
10	0.756	0.811	0.851	0.869	0.886	0.898	0.904	0.9091
20	0.796	0.865	0.906	0.922	0.935	0.943	0.948	0.9524
30	0.816	0.889	0.928	0.943	0.947	0.961	0.964	0.9677
40	0.826	0.903	0.942	0.955	0.963	0.971	0.974	0.9756
50	0.834	0.914	0.951	0.962	0.97	0.975	0.978	0.9804
60	0.844	0.92	0.957	0.968	0.974	0.98	0.982	0.9836
90	0.862	0.935	0.969	0.977	0.982	0.986	0.987	0.989
100	0.867	0.939	0.971	0.979	0.984	0.987	0.989	0.9901
500	0.891	0.974	0.993	0.995	0.996	0.997	0.998	0.998

Download free eBooks at bookboon.com

Now, extract each column in to separate vectors:

$NTU\_0 := Regen\_M^{\langle 0 \rangle}$ $cm08 := Regen\_M^{\langle 1 \rangle}$ $cm1 := Regen\_M^{\langle 2 \rangle}$

$cm125 := Regen\_M^{\langle 3 \rangle}$ $cm150 := Regen\_M^{\langle 4 \rangle}$ $cm2 := Regen\_M^{\langle 5 \rangle}$

$cm3 := Regen\_M^{\langle 6 \rangle}$ $cm5 := Regen\_M^{\langle 7 \rangle}$ $cm10000000 := Regen\_M^{\langle 8 \rangle}$

Use Mathcad built-in function 'linterp' to interpolate:

$Effcy\_Cm08(NTU) := linterp(NTU\_0, cm08, NTU)$ $Effcy\_Cm08(100) = 0.867$

$Effcy\_Cm1(NTU) := linterp(NTU\_0, cm1, NTU)$ $Effcy\_Cm1(100) = 0.939$

$Effcy\_Cm125(NTU) := linterp(NTU\_0, cm125, NTU)$ $Effcy\_Cm125(100) = 0.971$

$Effcy\_Cm150(NTU) := linterp(NTU\_0, cm150, NTU)$ $Effcy\_Cm150(100) = 0.979$

Download free eBooks at bookboon.com
Click on the ad to read more

$Effcy\_Cm2(NTU) := linterp(NTU\_0, cm2, NTU)$ $Effcy\_Cm2(100) = 0.984$

$Effcy\_Cm3(NTU) := linterp(NTU\_0, cm3, NTU)$ $Effcy\_Cm3(100) = 0.987$

$Effcy\_Cm5(NTU) := linterp(NTU\_0, cm5, NTU)$ $Effcy\_Cm5(100) = 0.989$

$Effcy\_Cm10000000(NTU) := linterp(NTU\_0, cm10000000, NTU)$ $Effcy\_Cm10000000(100) = 0.99$

Now, write the desired Function to find effectiveness for any given NTU_0 and Cm, by linear interpolation:

$Regn\_effcy(NTU\_0, Cm) :=$

return "Cm should be between 0 and Infinity " if $Cm < 0 \vee Cm > 10000000$

return "NTU_0 should be between 0 and 500 " if $NTU\_0 < 0 \vee NTU\_0 > 500$

$effcy \leftarrow Effcy\_Cm08(NTU\_0) + \frac{(Cm - 0.8)}{(1 - 0.8)} \cdot (Effcy\_Cm1(NTU\_0) - Effcy\_Cm08(NTU\_0))$ if $Cm \geq 0.8 \wedge Cm < 1$

$effcy \leftarrow Effcy\_Cm1(NTU\_0) + \frac{(Cm - 1)}{(1.25 - 1)} \cdot (Effcy\_Cm125(NTU\_0) - Effcy\_Cm1(NTU\_0))$ if $Cm \geq 1 \wedge Cm < 1.25$

$effcy \leftarrow Effcy\_Cm125(NTU\_0) + \frac{(Cm - 1.25)}{(1.5 - 1.25)} \cdot (Effcy\_Cm150(NTU\_0) - Effcy\_Cm125(NTU\_0))$ if $Cm \geq 1.25 \wedge Cm < 1.5$

$effcy \leftarrow Effcy\_Cm150(NTU\_0) + \frac{(Cm - 1.5)}{(2 - 1.5)} \cdot (Effcy\_Cm2(NTU\_0) - Effcy\_Cm150(NTU\_0))$ if $Cm \geq 1.5 \wedge Cm < 2$

$effcy \leftarrow Effcy\_Cm2(NTU\_0) + \frac{(Cm - 2)}{(3 - 2)} \cdot (Effcy\_Cm3(NTU\_0) - Effcy\_Cm2(NTU\_0))$ if $Cm \geq 2 \wedge Cm < 3$

$effcy \leftarrow Effcy\_Cm3(NTU\_0) + \frac{(Cm - 3)}{(5 - 3)} \cdot (Effcy\_Cm5(NTU\_0) - Effcy\_Cm3(NTU\_0))$ if $Cm \geq 3 \wedge Cm < 5$

$effcy \leftarrow Effcy\_Cm5(NTU\_0) + \frac{(Cm - 5)}{(10000000 - 5)} \cdot (Effcy\_Cm10000000(NTU\_0) - Effcy\_Cm5(NTU\_0))$ if $Cm \geq 5 \wedge Cm < 10000000$

$\begin{pmatrix} "NTU\_0" & "Cm" & "effcy" \\ NTU\_0 & Cm & effcy \end{pmatrix}$

Ex:

$NTU\_0 := 4$ $Cm := 4$

$Regn\_effcy(NTU\_0, Cm) = \begin{pmatrix} "NTU\_0" & "Cm" & "effcy" \\ 4 & 4 & 0.792 \end{pmatrix}$

==========

Download free eBooks at bookboon.com

Prob.4.2.37 Using the NTU-effectiveness method, determine the effectiveness for a valved type regenerator having a matrix material with a sp. heat of 0.48 kJ/kg.K and a total mass of 2688 kg. The mass flow rate of gas during the heating period is 2 kg/s and the mass flow rate during the cooling period is 1.8 kg/s. The sp. heat of gas for both the periods is 1.006 kJ/kg.K. The total period for the regenerator is 9 min. The convective heat transfer coeff for the heating period is 416 W/m^2.K and that for the cooling period is 388 W/m^2.K. The surface area for the heating and cooling periods is equal to 108 m^2 for each period. [1].

Data:

ms := 2688 kg .... matrix mass

cs := 480 J/kg.K ... sp. heat of matrix

mdot_h := 2 kg/s ... mass fow rate of gas during heating period

mdot_c := 1.8 kg/s ... mass fow rate of gas during cooling period

cph := 1006 J/kg.K ... sp.heat of gas during heating period

cpc := 1006 J/kg.K ... sp.heat of gas during cooling period

P0 := 540 s.... total period

hh := 416 W/m^2.K ... heat tr coeff during heating period

hc := 388 W/m^2.K ... heat tr coeff during cooling period

Ah := 108 m^2 ... heat tr area during heating period

Ac := 108 m^2 ... heat tr area during cooling period

Calculations:

$C_c := mdot\_c \cdot cpc$ i.e. $C_c = 1.811 \times 10^3$ W/K

$C_h := mdot\_h \cdot cph$ i.e. $C_h = 2.012 \times 10^3$ W/K

Therefore, cold fluid is min. fluid. i.e.

$C_{min} := C_c$ $C_{max} := C_h$

and,

$C_R := \frac{C_{min}}{C_{max}}$ i.e. $C_R = 0.9$ ...capacity ratio

Download free eBooks at bookboon.com

Since CR is not equal to 1, it is an 'unbalanced' regenerator.

$$NTU_0 := \frac{1}{C_{min}} \cdot \left(\frac{1}{hh \cdot Ah} + \frac{1}{hc \cdot Ac} \right)^{-1} \qquad \text{i.e.} \quad NTU_0 = 11.974$$

$$C_m := \frac{ms \cdot cs}{P0 \cdot C_{min}} \qquad \text{i.e.} \quad C_m = 1.319 \quad \text{..matrix capacity rate ratio}$$

Calculate the 'effective' NTU and Cm:

$$NTU_e := \frac{2 \cdot NTU_0 \cdot C_R}{(1 + C_R)} \qquad \text{i.e.} \quad NTU_e = 11.343$$

$$C_{me} := \frac{2 \cdot C_m \cdot C_R}{(1 + C_R)} \qquad \text{i.e.} \quad C_{me} = 1.25$$

For these effective values, find the regen. effectiveness from the Table 5.2; i.e. in our case, from the Mathcad Function written above:

$$Regn\_effcy(NTU_e, C_{me}) = \begin{pmatrix} "NTU\_0" & "Cm" & "effcy" \\ 11.343 & 1.25 & 0.858 \end{pmatrix}$$

This e-book
is made with
SetaPDF

SETASIGN

PDF components for **PHP** developers

www.setasign.com

Download free eBooks at bookboon.com

Click on the ad to read more

i.e. we get:

$\varepsilon_1 := \text{Regn\_effcy}(NTU_e, C_{me})_{1,2}$

i.e. $\varepsilon_1 = 0.858$ **...effectiveness for balanced regenerator**

Now, find parameter, X:

$$X := \frac{1 + C_R}{2 \cdot C_R} \cdot \frac{\varepsilon_1}{1 - \varepsilon_1} \cdot (1 - C_R) \qquad \text{i.e.} \quad X = 0.64$$

Therefore, Regenerator effectiveness:

$$\varepsilon := \frac{1 - \exp(-X)}{1 - C_R \cdot \exp(-X)}$$

i.e. $\varepsilon = 0.8996$ **...effectiveness of regenerator .... Ans.**

==========

Prob.4.2.38 A regenerator matrix is made of stacked wire screens having an equivalent dia of 0.4 mm. The free flow area through the screens is 0.003 m^2. Porosity of matrix is 0.675. The mean temp of air flowing through the regenerator is 200 K, and the gas properties are: μ = 13.3 μ Pa.s, cp = 1.006 kJ/kg.K and Pr = 0.739. The mass flow rate of air through the regenerator = 50 g/s. Determine the convective heat transfer coeff between the air and matrix material. [1].

Also, plot the convective heat transfer coeff as the mass flow rate varies from 30 g/s to 100 g/s.

For stacked wire screens, we have, for Colburn j-factor and friction factor [1]:

$$n = 0.37 + \frac{0.0136 \cdot e_v}{1 - e_v} \qquad jH = \frac{0.2 \cdot Re^{-n}}{1 - e_v} \quad \text{...Colburn j-factor}$$

$$f = \frac{860 \cdot (1 - e_v)}{Re} + \frac{2.2 \cdot e_v}{Re^{0.1}} \quad \text{...friction factor}$$

Download free eBooks at bookboon.com

Data:

$d_e := 0.0004$ m .. equiv. dia

$A_{ff} := 3 \cdot 10^{-3}$ m^2 ... free flow area

$e_v := 0.675$ ..porosity of matrix

$Tm := 200$ K .... mean temp of air flowing through regen.

$mdot := 0.05$ kg/s .... mass flow rate of air

Gas props at Tm = 200 K:

$\mu := 13.3 \cdot 10^{-6}$ Pa.s .... viscosity

$cp := 1006$ J/kg.K ... sp. heat

$Pr := 0.739$ Prandtl No.

Calculations:

Mass velocity:

$$G := \frac{mdot}{A_{ff}}$$ i.e. $G = 16.667$ kg/m^2.s .... mass velocity

Reynolds Number:

$$Re := \frac{G \cdot d_e}{\mu}$$ i.e. $Re = 501.253$ **....Reynolds No.**

Colburn j-factor:

$$n := 0.37 + \frac{0.0136 \cdot e_v}{1 - e_v}$$

$$jH := \frac{0.2 \cdot Re^{-n}}{1 - e_v}$$

i.e. $jH = 0.052$ **...Colburn j-factor**

But, we have, by definition:

$$jH = \frac{h_c}{cp \cdot G} \cdot Pr^{\frac{2}{3}}$$

Download free eBooks at bookboon.com

Therefore, convective heat transfer coeff. hc:

$$hc := \frac{jH \cdot cp \cdot G}{Pr^{\frac{2}{3}}}$$

i.e. $hc = 1.061 \times 10^{3}$ **W/m^2.K ..... heat transfer coeff.... Ans.**

Plot the convective heat transfer coeff as the mass flow rate varies from 30 g/s to 100 g/s:

First, write the relevant quantities as functions of mdot:

$$G(mdot) := \frac{mdot}{A_{ff}}$$

$$Re(mdot) := \frac{G(mdot) \cdot d_e}{\mu}$$

Deloitte.

Discover the truth at www.deloitte.ca/careers

© Deloitte & Touche LLP and affiliated entities.

Download free eBooks at bookboon.com

Click on the ad to read more

$$jH(mdot) := \frac{0.2 \cdot Re(mdot)^{-n}}{1 - e_v}$$

$$hc(mdot) := \frac{jH(mdot) \cdot cp \cdot G(mdot)}{Pr^{\frac{2}{3}}}$$

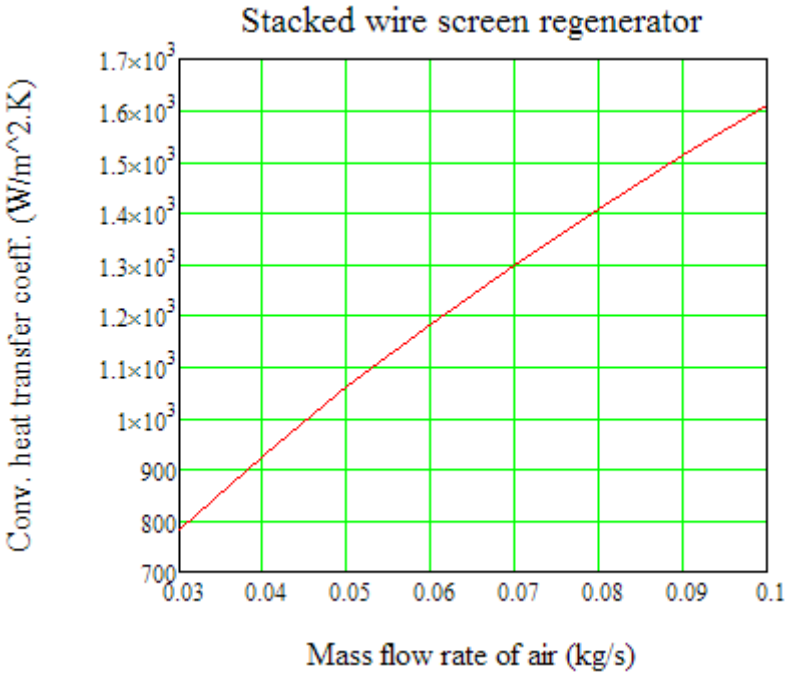
Now, define a range variable, mdot:

mdot := 0.03, 0.04 .. 0.1 ...define a range variable

And, we get:

mdot =	Re(mdot) =	jH(mdot) =	hc(mdot) =
0.03	300.752	0.063	780.509
0.04	401.003	0.057	928.027
0.05	501.253	0.052	$1.061 \cdot 10^3$
0.06	601.504	0.048	$1.184 \cdot 10^3$
0.07	701.754	0.045	$1.3 \cdot 10^3$
0.08	802.005	0.043	$1.408 \cdot 10^3$
0.09	902.256	0.041	$1.512 \cdot 10^3$
0.1	$1.003 \cdot 10^3$	0.039	$1.611 \cdot 10^3$

Plot:



Download free eBooks at bookboon.com

Prob.4.2.39 Nitrogen gas at an average temp of 200 K and an average pressure of 506.6 kPa flows through a regenerator matrix made of 3 mm dia random stacked spheres at a mass flow rate of 20 g/s. Porosity of matrix is 0.4 and the matrix is contained in a tube having an inside dia of 80 mm and a length of 750 mm. The properties of nitrogen gas are: μ = 12.95 μ Pa.s, ρ = 8.56 kg/m^3. Determine the pressure drop of nitrogen flowing through the regenerator. [1].

Also, plot ΔP vs mdot as mdot varies from 20 g/s to 70 g/s.

For random stacked spheres, we have, for Colburn j-factor and friction factor [1]:

$$jH = \frac{h_c \cdot Pr^{\frac{2}{3}}}{G \cdot cp} = 0.255 \cdot (e_v \cdot Re)^{\frac{-1}{3}} \qquad G = \frac{mdot}{A_{ff}} \qquad A_{ff} = \frac{e_v \cdot V_0}{L}$$

$$f = \frac{1 - e_v}{e_v} \cdot \left[\frac{570 \cdot (1 - e_v)^2}{e_v \cdot Re} + 3.5 \right] \qquad A = \frac{6 \cdot (1 - e_v) \cdot V_0}{D_s}$$

$$\Delta P = \left(\frac{f \cdot L}{D_s} \right) \cdot \left(\frac{G^2}{2 \cdot \rho} \right)$$

Data:

$D_s := 0.003$ m.. dia of spheres

$mdot := 0.02$ kg/s ... mass flow rate of nitrogen gas

$e_v := 0.4$ ....porosity

$d_i := 0.08$ m ... ID of tube

$L := 0.75$ m .. Length of tube

Gas properties:

$\mu := 12.95 \cdot 10^{-6}$ Pa.s .... dyn. viscosity

$\rho := 8.56$ kg/m^3 .... density of gas

Calculations:

Tube volume:

$V_0 := \frac{\pi \cdot d_i^2}{4} \cdot L$ i.e. $V_0 = 3.77 \times 10^{-3}$ m^3

Download free eBooks at bookboon.com

Free flow area:

$$A_{ff} := \frac{e_v \cdot V_0}{L} \qquad \text{i.e.} \quad A_{ff} = 2.011 \times 10^{-3} \quad \text{m^2}$$

Mass velocity:

$$G := \frac{mdot}{A_{ff}} \qquad \text{i.e.} \quad G = 9.947 \quad \text{kg/s.m^2}$$

Reynolds Number:

$$Re := \frac{G \cdot D_s}{\mu} \qquad \text{i.e.} \quad Re = 2.304 \times 10^{3} \quad \text{... Reynolds No.}$$

Friction factor:

$$f := \frac{1 - e_v}{e_v} \cdot \left[\frac{570 \cdot (1 - e_v)^2}{e_v \cdot Re} + 3.5 \right] \qquad \text{i.e.} \quad f = 5.584 \quad \text{...friction factor}$$

be
your degree
© 2013 Accenture. All rights reserved.
Bring your talent and passion to a global organization at the forefront of business, technology and innovation. Discover how great you can be.
Visit accenture.com/bookboon
Be greater than.
consulting | technology | outsourcing
accenture
High performance. Delivered.

Download free eBooks at bookboon.com

Click on the ad to read more

Pressure drop:

$$\Delta P := \left(\frac{f \cdot L}{D_s}\right) \cdot \left(\frac{G^2}{2 \cdot \rho}\right)$$

i.e. $\Delta P = 8.068 \times 10^3$ **Pa .... Ans.**

Plot ΔP as mdot varies from 20 g/s to 70 g/s:

First, write relevant quantities as functions of mdot:

$$G(mdot) := \frac{mdot}{A_{ff}}$$

$$Re(mdot) := \frac{G(mdot) \cdot D_s}{\mu}$$

$$f(mdot) := \frac{1 - e_v}{e_v} \cdot \left[\frac{570 \cdot (1 - e_v)^2}{e_v \cdot Re(mdot)} + 3.5\right]$$

$$\Delta P(mdot) := \left(\frac{f(mdot) \cdot L}{D_s}\right) \cdot \left(\frac{G(mdot)^2}{2 \cdot \rho}\right)$$

Now, define a range variable for mdot:

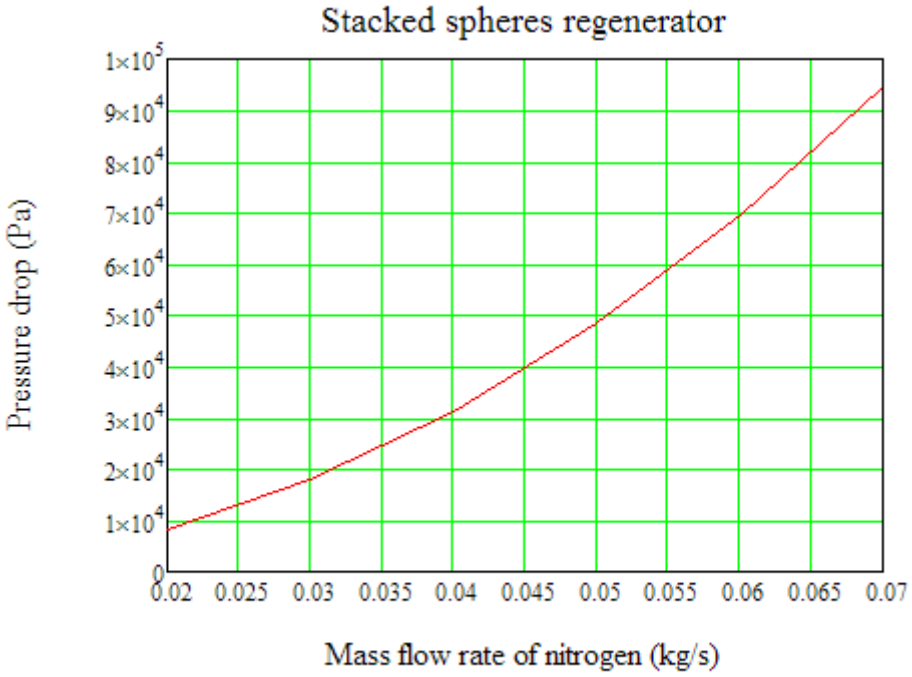
$mdot := 0.02, 0.03 .. 0.07$ ... define a range variable

We get:

mdot =	Re(mdot) =	f(mdot) =	ΔP(mdot) =
0.02	$2.304 \cdot 10^3$	5.584	$8.068 \cdot 10^3$
0.03	$3.457 \cdot 10^3$	5.473	$1.779 \cdot 10^4$
0.04	$4.609 \cdot 10^3$	5.417	$3.131 \cdot 10^4$
0.05	$5.761 \cdot 10^3$	5.384	$4.862 \cdot 10^4$
0.06	$6.913 \cdot 10^3$	5.361	$6.972 \cdot 10^4$
0.07	$8.065 \cdot 10^3$	5.345	$9.461 \cdot 10^4$

Download free eBooks at bookboon.com

Plot:



==========

Prob.4.2.40 Find an expression for the magnetocaloric coeff when the Curie law is valid.

Solution:

We have the Curie law:

$$I=\frac{C\cdot H}{T}$$

where C = Curie const.(K-m^3/kg) H = magn. field (A/m), I = magn. moment per unit mass (A-m^2/kg),

By definition:

Magneto-caloric effect:

$$\mu_m=\left(\frac{d}{dH}T\right)_s$$

Then, we get:

$$\mu_m=\frac{-\mu_0\cdot T}{C_H}\cdot\left(\frac{d}{dT}I\right)_H=\frac{\mu_0\cdot I}{C_H}$$ ...Ans.

Download free eBooks at bookboon.com

Prob.4.2.41 If a paramagnetic material obeys the modified Curie Law, find an expression for the magnetocaloric coeff.

Modified or corrected form of Curie Law is given below:

$$I=\frac{C\cdot H}{T}-\frac{D\cdot H^3}{T^3}$$ where D is the correction const.

Then, we have:

$$\mu_M=\frac{-\mu_0\cdot T}{C_H}\cdot\left(\frac{d}{dT}I\right)_H$$

where:

$\mu_0 := 4\cdot\pi\cdot 10^{-7}$ T-m/A ... permeability of free space

And: $\left(\frac{d}{dT}I\right)_H=\left(-C\cdot\frac{H}{T^2}+3\cdot D\cdot\frac{H^3}{T^4}\right)$

The Wake
the only emission we want to leave behind
Low-speed Engines Medium-speed Engines Turbochargers Propellers Propulsion Packages PrimeServ
The design of eco-friendly marine power and propulsion solutions is crucial for MAN Diesel & Turbo.
Power competencies are offered with the world's largest engine programme – having outputs spanning
from 450 to 87,220 kW per engine. Get up front!
Find out more at www.mandieselturbo.com
Engineering the Future – since 1758.
MAN Diesel & Turbo
MAN
Download free eBooks at bookboon.com
Click on the ad to read more

Therefore:

$$\mu_M = \frac{-\mu_0 \cdot T}{C_H} \cdot \left\langle -C \cdot \frac{H}{T^2} + 3 \cdot D \cdot \frac{H^3}{T^4} \right\rangle = \frac{\mu_0}{C_H} \cdot \left\langle 1 - \frac{2 \cdot D \cdot H^3}{T^3} \right\rangle \quad \text{....Ans.}$$

==========

Prob.4.2.42 Write a Mathcad Function for Brillouin function and entropy as a function of Brillouin constant ξ and atomic constant J. Tabulate the results and draw the graphs.

Solution:

Brillouin function and entropy for paramagnetic materials:

Brillouin function:

$$B(\xi, J) := (2 \cdot J + 1) \cdot \coth((2 \cdot J + 1) \cdot \xi) - \coth(\xi)$$

where

$$\xi = \frac{g \cdot \mu_B \cdot \mu_0 \cdot H}{2 \cdot k \cdot T} \quad \text{... Brillouin dimensionless parameter}$$

Ex: $J := \frac{5}{2}$ $\xi := 0.1$ We get: $B(\xi, J) = 1.13884$

Entropy:

$$\frac{s}{R} = \ln(2 \cdot J + 1) \quad \text{...for H = 0}$$

$$sbyR(\xi, J) := \ln\left[\frac{\sinh((2 \cdot J + 1) \cdot \xi)}{\sinh(\xi)}\right] - \xi \cdot B(\xi, J)$$

Ex: $sbyR(\xi, J) = 1.73551$

Download free eBooks at bookboon.com

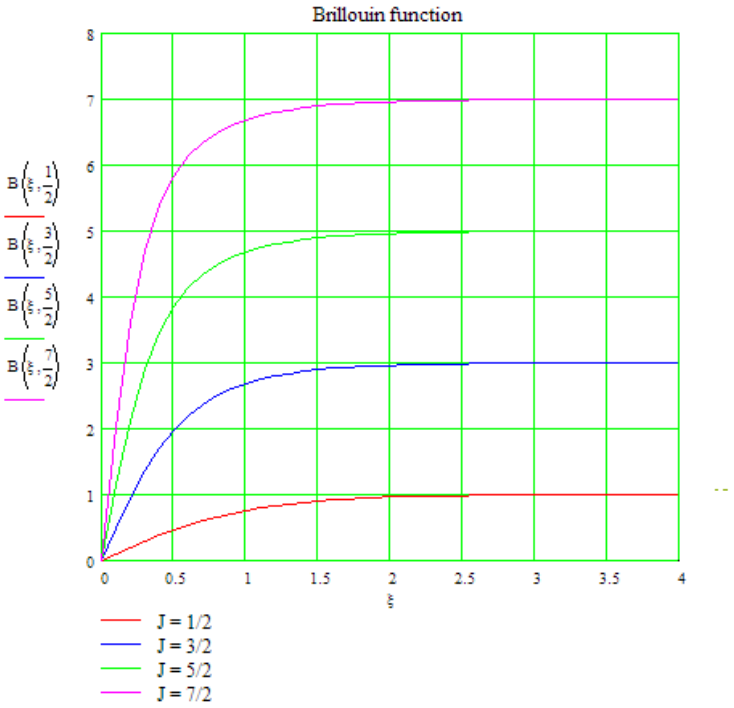
To produce the Table:

ξ	$B\left(\xi,\frac{1}{2}\right)$	$sbyR\left(\xi,\frac{1}{2}\right)$	$B\left(\xi,\frac{3}{2}\right)$	$sbyR\left(\xi,\frac{3}{2}\right)$	$B\left(\xi,\frac{5}{2}\right)$	$sbyR\left(\xi,\frac{5}{2}\right)$	$B\left(\xi,\frac{7}{2}\right)$	$sbyR\left(\xi,\frac{7}{2}\right)$
$1\cdot10^{-6}$	0	0.69315	0.00001	1.38629	0.00001	1.79176	0.00002	2.07944
0.1	0.09967	0.68817	0.49442	1.36171	1.13884	1.73551	2.01421	1.98083
0.2	0.19737	0.67354	0.95727	1.29266	2.13073	1.58839	3.61341	1.74534
0.3	0.29131	0.65009	1.3654	1.19115	2.90434	1.39686	4.70001	1.4776
0.4	0.37995	0.61912	1.70801	1.0718	3.46763	1.2013	5.39468	1.23708
0.5	0.46211	0.5822	1.9853	0.94754	3.86585	1.02327	5.8414	1.03764
0.6	0.53705	0.54105	2.20435	0.82751	4.14693	0.86948	6.13905	0.87489
0.7	0.60436	0.49736	2.37506	0.71691	4.34807	0.73928	6.34559	0.74123
0.8	0.66403	0.45267	2.50737	0.61796	4.49486	0.62956	6.49409	0.63024
0.9	0.71629	0.40831	2.6099	0.53102	4.60417	0.53691	6.60393	0.53714
1	0.76159	0.36534	2.68964	0.45543	4.68703	0.45838	6.68696	0.45845
1.1	0.8005	0.32454	2.75198	0.39011	4.7508	0.39156	6.75077	0.39159
1.2	0.83365	0.28645	2.801	0.33383	4.80046	0.33454	6.80046	0.33455
1.3	0.86172	0.25141	2.83977	0.28544	4.83953	0.28578	6.83953	0.28579
1.4	0.88535	0.21954	2.87061	0.24387	4.8705	0.24403	6.8705	0.24404
1.5	0.90515	0.19087	2.89525	0.20818	4.89521	0.20826	6.89521	0.20826
1.6	0.92167	0.16529	2.91503	0.17756	4.91501	0.1776	6.91501	0.1776
1.7	0.93541	0.14264	2.93096	0.15132	4.93095	0.15133	6.93095	0.15133
1.8	0.9468	0.12271	2.94382	0.12883	4.94382	0.12884	6.94382	0.12884
1.9	0.95624	0.10528	2.95423	0.10958	4.95423	0.10958	6.95423	0.10958
2	0.96403	0.0901	2.96268	0.09312	4.96268	0.09312	6.96268	0.09312
2.5	0.98661	0.04018	2.98643	0.04068	4.98643	0.04068	6.98643	0.04068
3	0.99505	0.01731	2.99503	0.01739	4.99503	0.01739	6.99503	0.01739
3.5	0.99818	0.00729	2.99817	0.0073	4.99817	0.0073	6.99817	0.0073
4	0.99933	0.00302	2.99933	0.00302	4.99933	0.00302	6.99933	0.00302

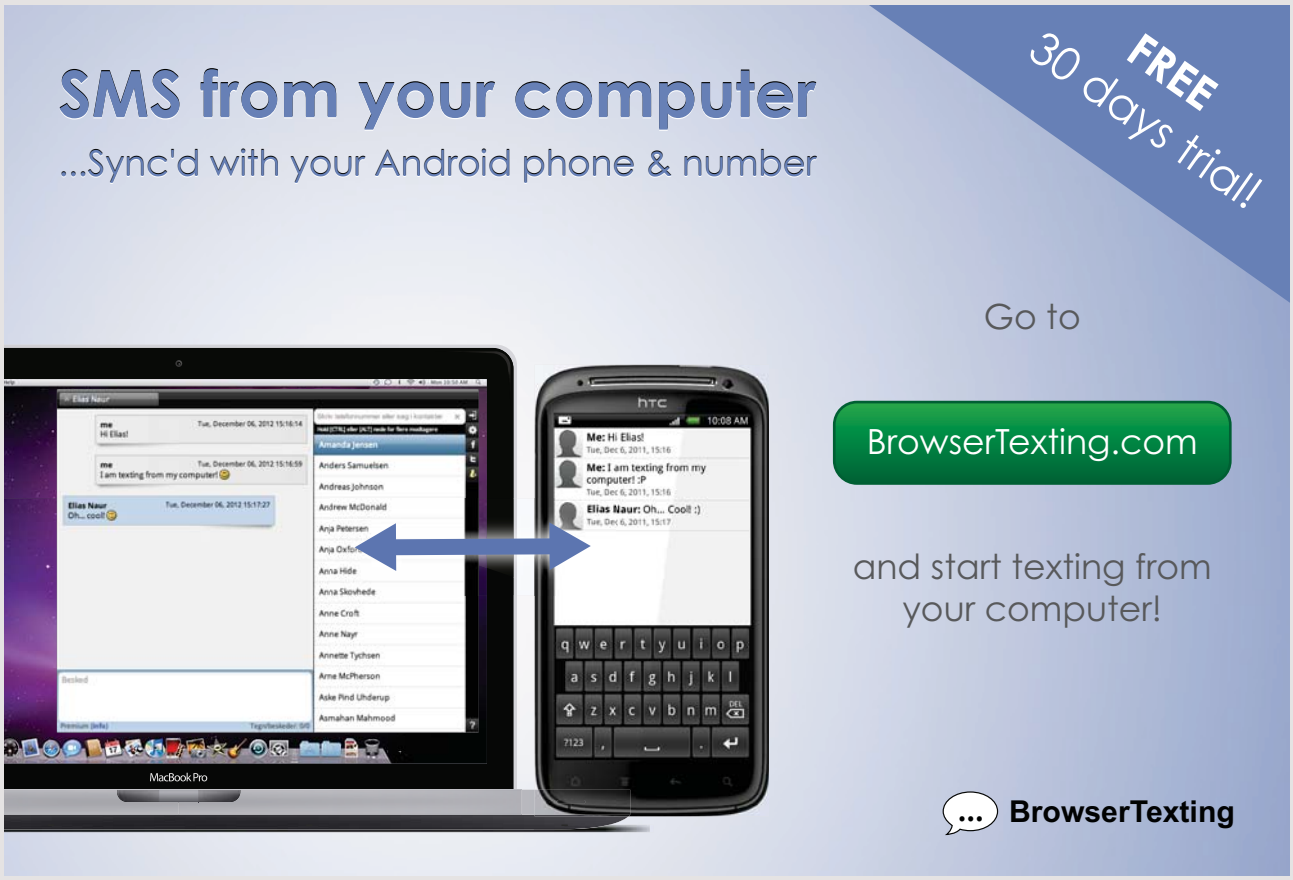
Plot the Brillouin function:

$\xi := 0.000001, 0.1 .. 4$ ...define a range variable

Download free eBooks at bookboon.com



:==========



Download free eBooks at bookboon.com

Click on the ad to read more

Prob.4.2.43 Write a simple Mathcad program to find the various properties of a given paramagnetic material. Use the data given in Ref.[1]

First, put all the properties in a Matrix, MM:

$$MM := \begin{array}{c|cccccc} & & g & J & M(g/mol) & R(J/kg.K) & C(K\text{-}m3/kg) \\ \hline & \text{"Cerium ethyl sulfate"} & 1.7 & \frac{2}{2} & 678 & 12.263 & 25.1\cdot10^{-6} \\ & \text{"Cerium magnesium nitrate"} & 1.84 & \frac{1}{2} & 765 & 10.868 & 5.22\cdot10^{-6} \\ & \text{"Chromium methylammonium alum"} & 2.0 & \frac{3}{2} & 492 & 16.899 & 30.5\cdot10^{-6} \\ & \text{"Chromium potassium alum"} & 1.97 & \frac{3}{2} & 499 & 16.662 & 45.8\cdot10^{-6} \\ & \text{"Copper potassium sulfate"} & 2.14 & \frac{1}{2} & 442 & 18.811 & 12.21\cdot10^{-6} \\ \hline & \text{"Gadolinium sulfate"} & 1.992 & \frac{7}{2} & 373 & 22.290 & 263.3\cdot10^{-6} \\ & \text{"Iron ammonium alum"} & 2.0 & \frac{5}{2} & 482 & 17.250 & 114.1\cdot10^{-6} \\ & \text{"Manganese ammonium sulfate"} & 2.06 & \frac{5}{2} & 391 & 21.264 & 149.2\cdot10^{-6} \\ & \text{"Titanium cesium alum"} & 1.89 & \frac{1}{2} & 589 & 14.116 & 4.21\cdot10^{-6} \end{array}$$

Now, write the Mathcad Program to find properties of a given paramagnetic material:

Inputs:

MM is a Matrix containing the names of materials and various properties viz.

Lande splitting factor, g,
Ionic const. J,
Ionic wt., M (g/mol),
Gas const. R, (J/kg.K) and
Curie const., C (K-m3/kg)

Download free eBooks at bookboon.com

k is the key which denotes different paramagnetic materials as follows:

k = 0: Cerium ethyl sulfate, 1 = Cerium magnesium nitrate, 2 = Chromium methylammonium alum, 3 = Chromium potassium alum, 4 = Copper potassium sulfate, 5 = Gadolinium sulfate, 6 = Iron ammonium alum, 7 = Manganese ammonium sulfate, 8 = Titanium cesium alum

Output:

Matrix containing various properties for the desired paramagnetic material.

Mathcad program:

Props_paramag(MM, k) :=

for m∈ 0.. 8

"Material"	"Lande splitting factor, g"	"Ionic const., J"	"Ionic wt., M (g/mol)"	"Gas const., R(J/kg.K)"	"Curie const. C (K-m3/kg)"
$MM_{k,0}$	$MM_{k,1}$	$MM_{k,2}$	$MM_{k,3}$	$MM_{k,4}$	$MM_{k,5}$

if k=m

Ex:

Props_paramag(MM, 1) =

"Material"	"Lande splitting factor, g"	"Ionic const., J"	"Ionic wt., M (g/mol)"	"Gas const., R(J/kg.K)"	"Curie const. C (K-m3/kg)"
"Cerium magnesium nitrate"	1.84	0.5	765	10.868	$5.22 \cdot 10^{-6}$

Prob.4.2.44 Determine the magnetic moment per unit mass and entropy for gadolinium sulfate at 0.1 K in a magnetic field of 60 kA/m (75.4 mT) assuming that Brillouin expression is valid at this condition.[1]

Solution:

We have, for Magnetic moment:

$I = (1/2).n.g.\ \mu_B.B(\xi)$

For gadolinium sulfate, we have following data, using the Function written above:

Props_paramag(MM, 5) =

"Material"	"Lande splitting factor, g"	"Ionic const., J"	"Ionic wt., M (g/mol)"	"Gas const., R(J/kg.K)"	"Curie const. C (K-m3/kg)"
"Gadolinum sulfate"	1.992	3.5	373	22.29	$2.633 \cdot 10^{-4}$

Download free eBooks at bookboon.com

Ionic weight, M = 0.373 kg/mol

Lande splitting factor, g = 1.992

Total angular momentum quantum no. J = 7/2

Also:

Avogadro's No. = N_0 = 6.023 x 10^{23} / mol

μ_B = Bohr magneton = 0.9273 x 10^{-23} A-m^2

μ_0 = permeability of free space = 4 π . 10^{-7} J/A-m^2

k = Boltzmann constant = k = 1.3805 x 10^{-23} J/K

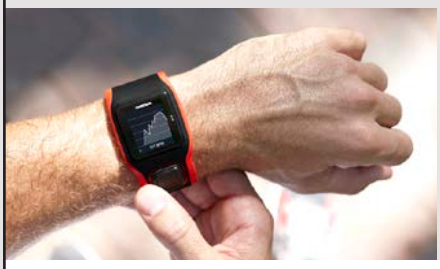
n = no. of ions per unit mass = N_0 / M

T = Temp (K)

YOUR WORK AT TOMTOM WILL BE TOUCHED BY MILLIONS. AROUND THE WORLD. EVERYDAY.

Join us now on www.TomTom.jobs

follow us on LinkedIn



#ACHIEVEMORE

TOMTOM

Download free eBooks at bookboon.com

Click on the ad to read more

Now, we have, Brillouin parameter:

$\xi = g.\mu_B.\ \mu_0.H\ /\ (2.k.T)$

Performing the calculations in Mathcad, we have:

$\xi = \frac{g \cdot \mu_B \cdot \mu_0 \cdot H}{2 \cdot k \cdot T}$ ... Brillouin dimensionless parameter

and: $g := 1.992$ ...Lande splitting factor for gadolinium sulfate

$J := \frac{7}{2}$ ...Lande splitting factor for gadolinium sulfate

$M := 0.373$ kg/mol ... ionic weight (or mol. wt.)

$\mu_B := 0.9273 \cdot 10^{-23}$ ...A-m^2 ... Bohr magneton

$\mu_0 := 4 \cdot \pi \cdot 10^{-7}$ ...J/A-m^2 ... permeability of free space

$k := 1.3805 \cdot 10^{-23}$ ...J/K ... Boltzmann const.

$H := 60 \cdot 10^3$ A/m ... by data

$T := 0.1$ K ... by data

$N_0 := 6.023 \cdot 10^{23}$ mol^-1 ... Avogadro's No.

Then, we get:

$\xi := \frac{g \cdot \mu_B \cdot \mu_0 \cdot H}{2 \cdot k \cdot T}$ ... Brillouin dimensionless parameter

i.e. $\xi = 0.504$

And, Brillouin function is calculated using the Mathcad Function written earlier:

$B(\xi, J) = 5.8572$

Now, no. of ions per unit mass:

$n := \frac{N_0}{M}$ i.e. $n = 1.6147 \cdot 10^{24}$ ions/kg

Therefore, magnetic moment:

$I := \frac{1}{2} \cdot n \cdot g \cdot \mu_B \cdot B(\xi, J)$

i.e. $I = 87.352$ **A-m^2/kg ... Ans.**

Download free eBooks at bookboon.com

To find entropy:

For gadolinium sulfate, we have:

R := 22.29 J/kg-K ..... Gas constant.

And, find (s/R) using the Mathcad Function written earlier:

sbyR(ξ, J) = 1.03

Then: s := sbyR(ξ, J)·R

i.e. s = 22.952 **J/kg-K ..... entropy ... Ans.**

==========

Prob.4.2.45 Determine the magnetic moment per unit mass and entropy for cerium magnesium nitrate at 0.1 K in a magnetic field of 189 kA/m (0.2375 T) assuming that Brillouin expression is valid at this condition. What would be the value of magnetic moment per unit mass according to the Curie law?[1]

Solution:

We have:

$$\xi = \frac{g \cdot \mu_B \cdot \mu_0 \cdot H}{2 \cdot k \cdot T}$$ ... Brillouin dimensionless parameter

And for CMN, we have:

Props_paramag(MM, 1) =

"Material"	"Lande splitting factor, g"	"Ionic const., J"	"Ionic wt., M (g/mol)"	"Gas const., R(J/kg.K)"	"Curie const. C (K-m3/kg)"
"Cerium magnesium nitrate"	1.84	0.5	765	10.868	$5.22 \cdot 10^{-6}$

Download free eBooks at bookboon.com

$g := 1.84$ ...Lande splitting factor for cerium magnesium nitrate

$J := \frac{1}{2}$ ...Lande splitting factor for cerium magnesium nitrate

$M := 0.765$ kg/mol ... ionic weight (or mol. wt.)

$\mu_B := 0.9273 \cdot 10^{-23}$ ...A-m^2 ... Bohr magneton

$\mu_0 := 4 \cdot \pi \cdot 10^{-7}$ ...J/A-m^2 ... permeability of free space

$k := 1.3805 \cdot 10^{-23}$ ...J/K ... Boltzmann const.

$H := 189 \cdot 10^{3}$ A/m ... by data

$T := 0.1$ K ... by data

$N_0 := 6.023 \cdot 10^{23}$ mol^-1 ... Avogadro's No.

Then, we get:

$$\xi := \frac{g \cdot \mu_B \cdot \mu_0 \cdot H}{2 \cdot k \cdot T}$$ ... Brillouin dimensionless parameter

i.e. $\xi = 1.468$

Brain power
By 2020, wind could provide one-tenth of our planet's electricity needs. Already today, SKF's innovative know-how is crucial to running a large proportion of the world's wind turbines.
Up to 25 % of the generating costs relate to maintenance. These can be reduced dramatically thanks to our systems for on-line condition monitoring and automatic lubrication. We help make it more economical to create cleaner, cheaper energy out of thin air.
By sharing our experience, expertise, and creativity, industries can boost performance beyond expectations.
Therefore we need the best employees who can meet this challenge!
The Power of Knowledge Engineering
Plug into The Power of Knowledge Engineering.
Visit us at www.skf.com/knowledge
SKF

Download free eBooks at bookboon.com

Click on the ad to read more

And, Brillouin function is calculated using the Mathcad Function written earlier:

$B(\xi, J) = 0.8991$

Now, no. of ions per unit mass:

$n := \frac{N_0}{M}$ i.e. $n = 7.8732 \cdot 10^{23}$ ions/kg

Therefore, magnetic moment:

$I := \frac{1}{2} \cdot n \cdot g \cdot \mu_B \cdot B(\xi, J)$

i.e. $I = 6.039$ **A-m^2/kg ... Ans.**

To find entropy:

For cerium magnesium nitrate, we have:

$R := 10.868$ J/kg-K ..... Gas constant.

And, find (s/R) using the Mathcad Function written earlier:

$sbyR(\xi, J) = 0.2$

Then: $s := sbyR(\xi, J) \cdot R$

i.e. $s = 2.171$ **J/kg-K ..... entropy ... Ans.**

If the Curie law is valid:

Curie const. for cerium magnesium nitrate is:

$C := 5.22 \cdot 10^{-6}$ K-m^3/kg

Using the Curie law, the magnetic moment is:

$I := \frac{C \cdot H}{T}$

i.e. $I = 9.866$ **A-m^2/kg ....Ans.**

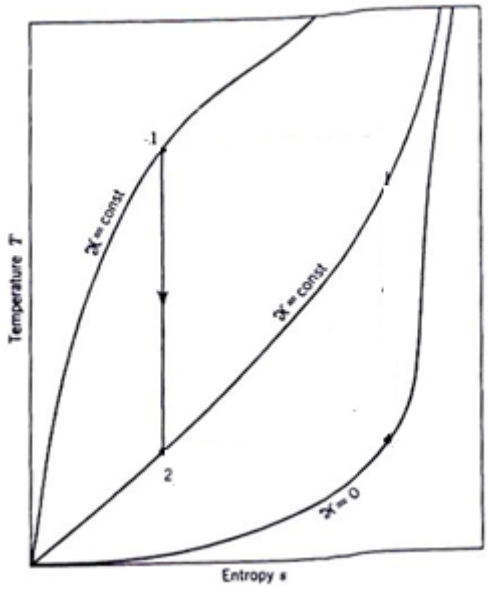
Comparing with the value obtained earlier, we see that the correct magnetic moment differs from the one obtained with Curie law by:

$\frac{(9.866 - 6.039) \cdot 100}{6.039} = 63.371$ **% ... Note that this is not negligible.**

==========

Download free eBooks at bookboon.com

Prob.4.2.46 Magnetic field intensity around a cerium magnesium nitrate (CMN) pellet is decreased reversibly and adiabatically from 320 kA/m (0.402 T) to 32 kA/m (40.2 mT). If the initial temp is 2 K, determine the final temp. [1]



Solution:

We have, for CMN:

$T1 := 2$ K $\quad H1 := 320 \cdot 10^{3}$ A/m $\quad H2 := 32 \cdot 10^{3}$ A/m $\quad J := \frac{1}{2}$

$R := 10.868$ J/kg-K ..... Gas constant.

$k := 1.3805 \cdot 10^{-23}$ ...J/K ... Boltzmann const.

Then:

$$\xi 1 := \frac{g \cdot \mu_B \cdot \mu_0 \cdot H1}{2 \cdot k \cdot T1}$$ ... Brillouin dimensionless parameter

i.e. $\xi 1 = 0.124$

And, find (s/R) using the Mathcad Function written earlier:

$sbyR(\xi, J) = 0.2$

Then: $s1 := sbyR(\xi 1, J) \cdot R$

i.e. $s1 = 7.45$ **J/kg-K ..... entropy before adiab. demagnetisation .**

Download free eBooks at bookboon.com

At the end of adiabatic demagnetisation: Let temp. be T2

Remember: entropy remains constant.

Using the 'Solve block' of Mathcad to find T2 with s2 = s1:

T2 := 0.1 K .... trial value

Given

$$s1 = sbyR\left(\frac{g \cdot \mu_B \cdot \mu_0 \cdot H2}{2 \cdot k \cdot T2}, J\right) \cdot R$$

T2 := Find(T2)

We get:

T2 = 0.2 **K ...final temp at the end of demagnetisation... Ans.**

==========

Download free eBooks at bookboon.com

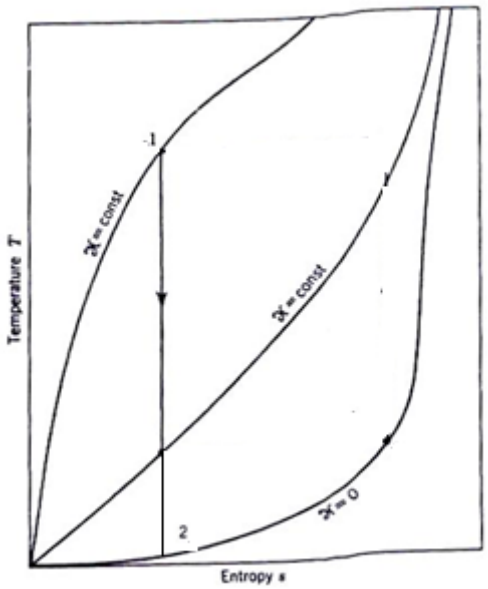
Click on the ad to read more

Prob.4.2.47 Magnetic field intensity around an iron ammonium alum pellet is decreased reversibly and adiabatically from 500 kA/m (0.628 T) to zero (external field). If the "internal field" for iron ammonium alum is 35 kA/m (44 mT), and the initial temp of the pellet is 1.5 K, determine the final temp. of the pellet. [1]

For Iron ammonium alum, from the Mathcad function written earlier:

Props_paramag(MM, 6) =

"Material"	"Lande splitting factor, g"	"Ionic const., J"	"Ionic wt., M (g/mol)"	"Gas const., R(J/kg.K)"	"Curie const. C (K-m3/kg)"
"Iron ammonium alum"	2	2.5	482	17.25	$1.141 \cdot 10^{-4}$



Solution:

We have, for Iron Ammonium alum:

$T1 := 1.5$ K $\quad H1 := 500 \cdot 10^3$ A/m $\quad H2 := 0$ A/m $\quad J := \frac{5}{2} \quad g := 2$

$R := 17.25$ J/kg-K ..... Gas constant. $\quad M := 482$ g/mol

$H\_int := 35 \cdot 10^3$ A/m .... internal magn. field

$k := 1.3805 \cdot 10^{-23}$ ...J/K ... Boltzmann const.

Download free eBooks at bookboon.com

Therefore: Effective magn. fields are:

$$H1\_eff := \left(H1^2 + H\_int^2\right)^{\frac{1}{2}} \quad \text{i.e.} \quad H1\_eff = 5.012 \cdot 10^5 \quad \text{A/m}$$

$$H2\_eff := \left(H2^2 + H\_int^2\right)^{\frac{1}{2}} \quad \text{i.e.} \quad H2\_eff = 3.5 \cdot 10^4 \quad \text{A/m}$$

Then:

$$\xi 1 := \frac{g \cdot \mu_B \cdot \mu_0 \cdot H1\_eff}{2 \cdot k \cdot T1} \quad \text{... Brillouin dimensionless parameter}$$

i.e. $\xi 1 = 0.282$

And, find (s/R) using the Mathcad Function written earlier:

Then: $s1 := sbyR(\xi 1, J) \cdot R$

i.e. $s1 = 24.71$ **J/kg-K ..... entropy before adiab. demagnetisation .**

At the end of adiabatic demagnetisation: Let temp. be T2

Remember: entropy remains constant.

Using the 'Solve block' of Mathcad to find T2 with s2 = s1:

$T2 := 0.2$ K .... trial value

Given

$$s1 = sbyR\left(\frac{g \cdot \mu_B \cdot \mu_0 \cdot H2\_eff}{2 \cdot k \cdot T2}, J\right) \cdot R$$

$T2 := Find(T2)$

We get:

$T2 = 0.105$ **K ...final temp at the end of demagnetisation... Ans.**

==========

Download free eBooks at bookboon.com

Prob.4.2.48 A magnetic refrigerator operates between 0 kA/m and 231.3 kA/m (0.291 T) during the reversible isothermal heat addition process. Heat is added to the salt from the low temp region at 0.22 K and heat is rejected to a helium bath at 2.5 K. The mass of the working salt is 25 g. and the refrigerator operates at 30 cycles per hour. Determine the heat transfer rate from the low temp source to the refrigerator, assuming the Brillouin expression is valid. The working salt is Chromium potassium alum. [1]

For Chromium potassium alum, using the Mathcad function written earlier:

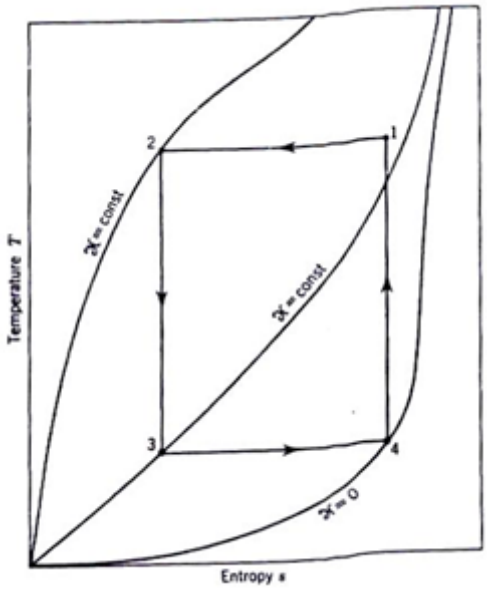
Props_paramag(MM, 3) =

"Material"	"Lande splitting factor, g"	"Ionic const., J"	"Ionic wt., M (g/mol)"	"Gas const., R(J/kg.K)"	"Curie const. C (K-m3/kg)"
"Chromium potassium alum"	1.97	1.5	499	16.662	$4.58 \cdot 10^{-5}$

LIGS University
based in Hawaii, USA
is currently enrolling in the
Interactive Online BBA, MBA, MSc,
DBA and PhD programs:
► enroll by October 31st, 2014 and
► save up to 11% on the tuition!
► pay in 10 installments / 2 years
► Interactive Online education
► visit www.ligsuniversity.com to find out more!
Note: LIGS University is not accredited by any nationally recognized accrediting agency listed by the US Secretary of Education.
More info here.

Download free eBooks at bookboon.com

Click on the ad to read more



Solution:

We have, for Chromium potassium alum:

$T4 := 0.22\ K \qquad H2 := 231.3 \cdot 10^{3}\ A/m \qquad H4 := 0\ A/m \qquad T2 := 2.5\ K$

$R := 16.662$ J/kg-K ..... Gas constant. $\qquad J := \frac{3}{2} \qquad g := 1.97$

$k := 1.3805 \cdot 10^{-23}$ ...J/K ... Boltzmann const.

$m := 0.025$ kg .... mass of pellet

Then:

$$\xi 2 := \frac{g \cdot \mu_B \cdot \mu_0 \cdot H2}{2 \cdot k \cdot T2}$$ ... Brillouin dimensionless parameter

i.e. $\xi 2 = 0.077$

And, find (s/R) using the Mathcad Function written earlier:

$sbyR(\xi 2, J) = 1.372$

Then: $s2 := sbyR(\xi 2, J) \cdot R$

i.e. $s2 = 22.854$ **J/kg-K ..... entropy before adiab. demagnetisation .**

Now: $s3 := s2$ **...for isentropic process 2-3**

Download free eBooks at bookboon.com

At zero magn. field, we have for entropy:

$s4 := R \cdot \ln(2 \cdot J + 1)$

i.e. $s4 = 23.098$ **J/kg-K ..... entropy after heat absorption at low temp.**

Therefore, heat transfer to refrigerator, Qpercycle:

$Qpercycle := m \cdot T4 \cdot (s4 - s3)$ J/cycle

i.e. $Qpercycle = 1.342 \cdot 10^{-3}$ J/cycle

And, heat transfer rate, Q for 30 cycles/h:

$Q := Qpercycle \cdot \frac{30}{3600}$ W

i.e. $Q = 1.118 \cdot 10^{-5}$ **W ... heat transfer rate at 0.22 K ... Ans.**

==========

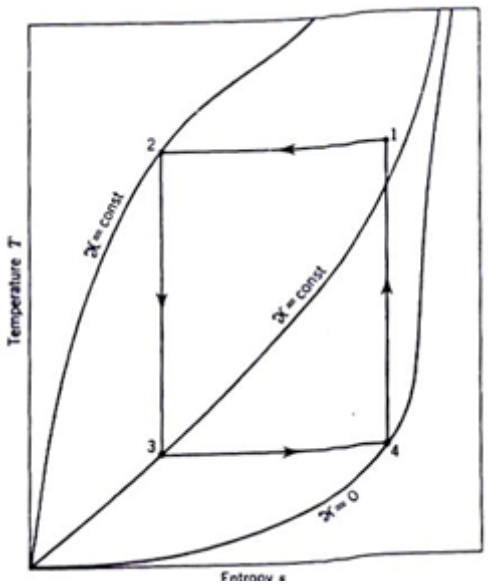
Prob.4.2.49 A magnetic refrigerator uses chromium methyl ammonium alum as the working salt. Heat addition per unit mass from low temp source at 0.08 K to the working salt is 1.258 J/kg. The magnetic field intensity at the end of isothermal heat addition process is 0 A/m. Determine the magnetic field intensity at the beginning of the isothermal heat addition process. [1]

For Chromium methyl ammonium alum, using the Mathcad function written earlier:

Props_paramag(MM, 2) =

"Material"	"Lande splitting factor, g"	"Ionic const., J"	"Ionic wt., M (g/mol)"	"Gas const., R(J/kg.K)"	"Curie const. C (K-m3/kg)"
"Chromium methylammonium alum"	2	1.5	492	16.899	$3.05 \cdot 10^{-5}$

Download free eBooks at bookboon.com



TURN TO THE EXPERTS FOR
SUBSCRIPTION CONSULTANCY

Subscrybe is one of the leading companies in Europe when it comes to innovation and business development within subscription businesses.

We innovate new subscription business models or improve existing ones. We do business reviews of existing subscription businesses and we develope acquisition and retention strategies.

Learn more at linkedin.com/company/subscrybe or contact
Managing Director Morten Suhr Hansen at mha@subscrybe.dk

SUBSCRYBE - to the future

Download free eBooks at bookboon.com
Click on the ad to read more

Solution:

We have, for Chromium methyl ammonium alum:

$T4 := 0.08\ K \qquad H4 := 0 \quad A/m \qquad T3 := T4$

$R := 16.899$ J/kg-K ..... Gas constant. $\qquad J := \frac{3}{2} \qquad g := 2$

$k := 1.3805 \cdot 10^{-23}$ ...J/K ... Boltzmann const.

$Qpercycle := 1.258$ J/kg

Then:

At zero magn. field, we have for entropy:

$$s4 := R \cdot \ln(2 \cdot J + 1)$$

i.e. $s4 = 23.427$ **J/kg-K ..... entropy after heat absorption at low temp.**

And, we have:

$$Qpercycle = T4 \cdot (s4 - s3) \qquad J/kg$$

Therefore: $s3 := s4 - \frac{Qpercycle}{T4}$

i.e. $s3 = 7.702$ J/kg.K

Now, knowing s3 and T3, find the magnetic field at point 3:

We have:

$$\xi 3 = \frac{g \cdot \mu_B \cdot \mu_0 \cdot H3}{2 \cdot k \cdot T3}$$ ... Brillouin dimensionless parameter

And, find (s/R) using the Mathcad Function written earlier.

Download free eBooks at bookboon.com

And, use the 'Solve block' of Mathcad to get H3:

$H3 := Find(H3)$

We get: $H3 = 9.473 \cdot 10^4$ A/m

i.e.

Magnetic field intensity at the beginning of isothermal heat addition process =

9.473 x 10^4 A/m.... Ans.

Prob.4.2.50 Nuclear paramagnetic materials obey the Brillouin expression, with the exception that the Bohr magneton must be replaced by the nuclear magneton (μN) = 5.0506 × 10-^{27} A-m^2 and the nuclear quantum number I replaces the atomic constant, J. Determine the magnetic moment per unit mass and entropy for copper as a nuclear paramagnetic system with g = 2, I = 1/2, M = 63.54 g/mol, and R = 130.8 J/kg.K. The copper temp is 0.004 K and the applied magnetic field is 4.35 MA/m (= 5.47 T).[1]

Solution:

We have:

$$\xi = \frac{g \cdot \mu_N \cdot \mu_0 \cdot H}{2 \cdot k \cdot T}$$ ... Brillouin dimensionless parameter

and:

$g := 2$ ...Lande splitting factor for copper

$I := \frac{1}{2}$ ...Nuclear quantum no. for copper

$M := 0.06354$ kg/mol ... ionic weight (or mol. wt.) for copper

$\mu_N := 5.0506 \cdot 10^{-27}$ ...A-m^2 ... Nuclear magneton

$\mu_0 := 4 \cdot \pi \cdot 10^{-7}$ ...J/A-m^2 ... permeability of free space

$k := 1.3805 \cdot 10^{-23}$ ...J/K ... Boltzmann const.

$H := 4.35 \cdot 10^6$ A/m ... by data

$T := 0.004$ K ... by data

$N_0 := 6.023 \cdot 10^{23}$ mol^-1 ... Avogadro's No.

Download free eBooks at bookboon.com

Then, we get:

$$\xi := \frac{g \cdot \mu_N \cdot \mu_0 \cdot H}{2 \cdot k \cdot T}$$ ... Brillouin dimensionless parameter

i.e. $\xi = 0.5$

And, Brillouin function is calculated using the Mathcad Function written earlier:

$B(\xi, I) = 0.4621$

Now, no. of ions per unit mass:

$$n := \frac{N_0}{M}$$ i.e. $n = 9.4791 \cdot 10^{24}$ ions/kg

Therefore, magnetic moment for copper:

$$I_{cu} := \frac{1}{2} \cdot n \cdot g \cdot \mu_N \cdot B(\xi, I)$$

i.e. $I_{cu} = 0.022$ **A-m^2/kg ... Ans.**

"I studied English for 16 years but... ...I finally learned to speak it in just six lessons"
Jane, Chinese architect
ENGLISH OUT THERE
Click to hear me talking before and after my unique course download

Download free eBooks at bookboon.com

Click on the ad to read more

To find entropy:

For copper, we have:

R := 130.8 J/kg-K ..... Gas constant.

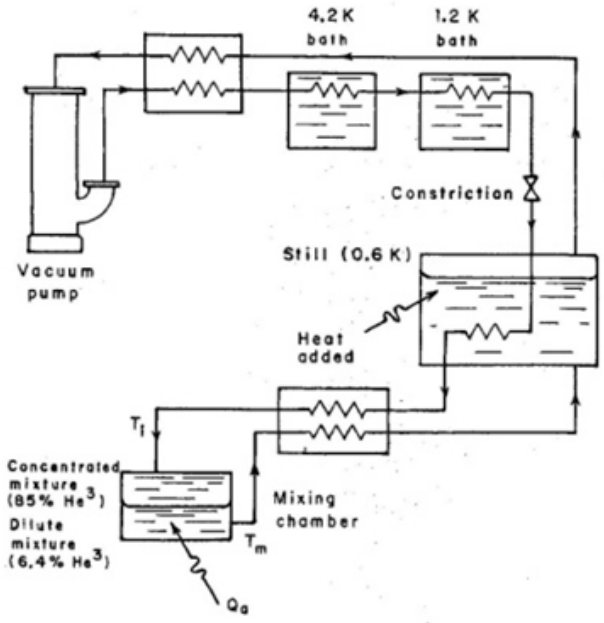
We have:

$$s := sbyR\left(\frac{g \cdot \mu_N \cdot \mu_0 \cdot H}{2 \cdot k \cdot T}, I\right) \cdot R$$

i.e. s = 76.154 **J/kg-K ..... entropy ... Ans.**

==========

Prob.4.2.51 A He3-He4 dilution refrigerator operates with a flow rate of He^3 of 1.53 x 10^{-4} mol/s. The liquid He^3 enters the mixing chamber at 0.04 K and the dilute phase leaves the mixing chamber at 0.03 K. Determine the heat transfer rate to the mixing chamber from the low temp region.[1]



Download free eBooks at bookboon.com

Solution:

$n_3 := 1.53 \cdot 10^{-4}$ mol/s

$T_i := 0.04$ K ... temp of He3 mixture entering the mixing chamber

$T_m := 0.03$ K ... temp of He3 mixture leaving the mixing chamber

$C_1 := 94$ J/mol.K^2 .

$C_2 := 12$ J/mol.K^2 .

Then, we have:

$h_m := C_1 \cdot T_m^2$ i.e. $h_m = 0.085$ J/mol

$h_i := C_2 \cdot T_i^2$ i.e. $h_i = 0.019$ J/mol

Therefore: heat transfer rate to the mixing chamber from low temp. region:

$Q_a := n_3 \cdot (h_m - h_i)$ W

i.e. $Q_a = 1.001 \cdot 10^{-5}$ **W = 10.01 micro-Watt .... Ans.**

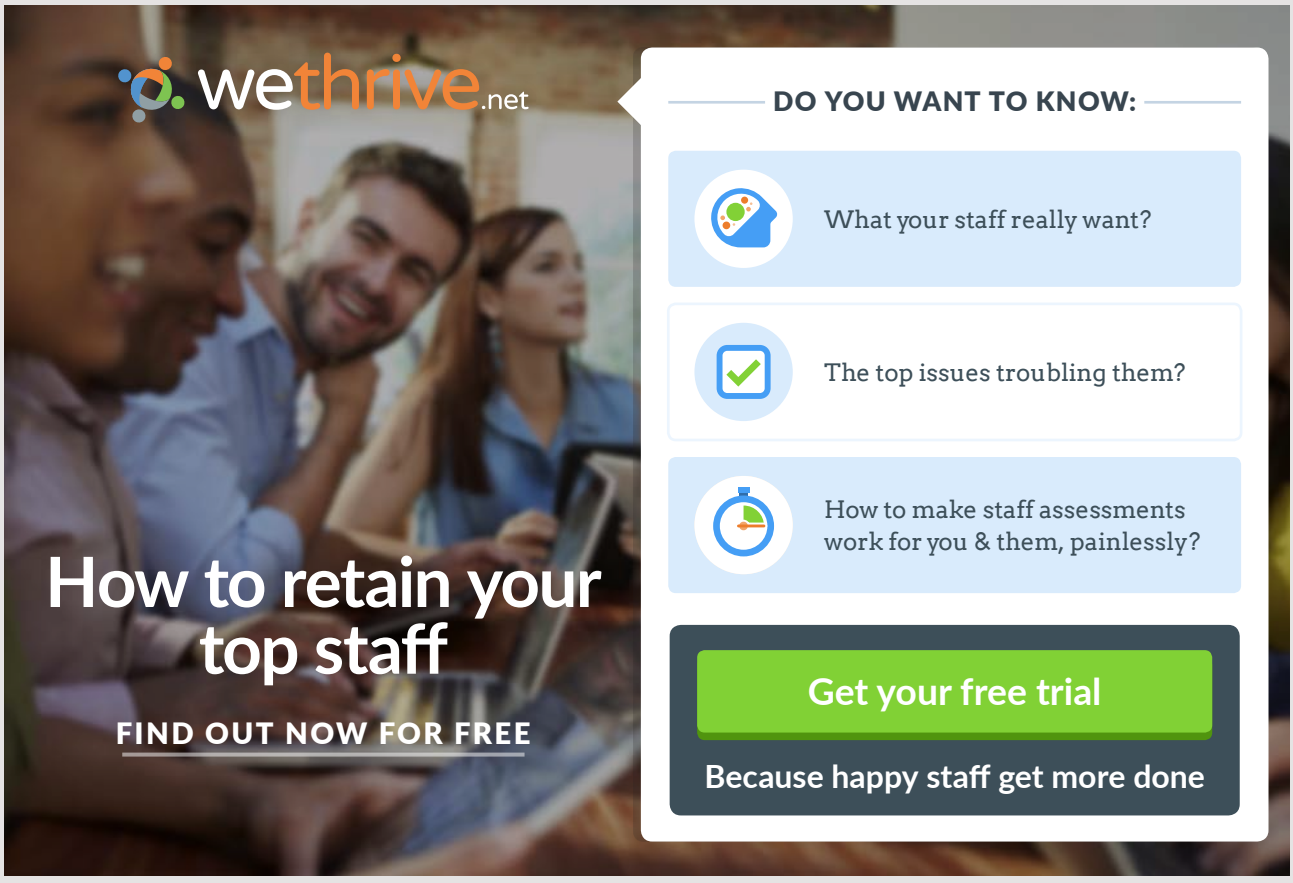
:==========

4.3 REFERENCES

1. *Randall F Barron*, Cryogenic Systems, McGraw Hill, 1985.
2. *Thomas Flynn*, Cryogenic Engineering, 2nd Ed. Marcel Dekker, 2005.
3. *Russel B Scot*, Cryogenic Engineering, D. Van Nostrand Co. Inc., 1959.
4. *GuglielmoVentura and Lara Risegari*, Art of Cryogenics – Low temp experimental techniques, Elsevier, 2008.
5. *White G K,* Experimental techniques in low temp physics, Oxford: Clarendon Press, 2nd Ed. 1968.
6. *S W Van Sciver,* Helium cryogenics, International cryogenics monograph series, Chapter 2, Springer Science + Business media.
7. https://en.wikipedia.org/wiki/Joule%E2%80%93Thomson_effect
8. https://en.wikipedia.org/wiki/Inversion_temperature
9. *J.M. Pfotenhauer,* University of Wisconsin, Madison, USPAS Cryogenics Short Course, Boston, 2010.
10. *J.G. Weisend II,* Refrigeration & liquefaction, in USPAS Cryogenics Short Course, 2013.
11. *Ph. Lebrun*, Introduction to Cryogenics, CERN, CAS & ALBA School on Vacuum in Accelerators Platja d'Aro, Spain, 16–24 May 2006.

Download free eBooks at bookboon.com

12. *M. Thirumaleshwar & S.V. Subramanyam,* Gifford – McMahon cycle – A theoretical analysis, Cryogenics, Vol. 26, March 1986.
13. *M. Thirumaleshwar & S.V. Subramanyam,* Heat balance analysis of single stage Gifford – McMahon cycle cryorefrigerator, Cryogenics, Vol. 26, March 1986.
14. *M. Thirumaleshwar & S.V. Subramanyam,* Two stage Gifford – McMahon cycle cryorefrigerator operating at 20 K, Cryogenics, Vol. 26, October 1986.
15. https://en.wikipedia.org/wiki/Magnetic_refrigeration.
16. http://www.cryomagnetics.com/products/nuclear-demagnetization-magnets/
17. http://www.americanmagnetics.com/ndm.php
18. *A. Arkharov et al.* Theory and Design of Cryogenic systems, Mir Publishers, Moscow, 1981.
19. https://www.nmri.go.jp/eng/khirata/stirling/schmidt/schmidt_e.pdf
20. *M. Thirumaleshwar*, Cryogenic Engineering: Software Solutions – Vol II B- Liquefaction systems + Problems (Mathcad), Bookboon.com, 2016.


wethrive.net
How to retain your top staff
FIND OUT NOW FOR FREE
DO YOU WANT TO KNOW:
What your staff really want?
The top issues troubling them?
How to make staff assessments work for you & them, painlessly?
Get your free trial
Because happy staff get more done

Download free eBooks at bookboon.com
Click on the ad to read more

To see Part III-B download:
Cryogenic Engineering: Software Solutions Part-III-B

Download free eBooks at bookboon.com